1. Let $b(n)$ denote the binary representation of $n$ without leading 0s (e.g., $b(10) = 1010$). Let $b(n)^R$ denote the reversal of such a string.

Show that $A = \{b(n)^R \# b(n+1) \mid n \geq 1\}$ is not a regular language. Provide two proofs of this fact, one using the Myhill-Nerode theorem and one using the Pumping Lemma.

2. Consider the DFA $D = (\{1, \ldots, 8\}, \{a, b\}, \delta, 1, \{1, 8\})$ with $\delta$ described by the following table:

<table>
<thead>
<tr>
<th>$q$</th>
<th>$\delta(q, a)$</th>
<th>$\delta(q, b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

Using the algorithm described in class:

(a) Determine which states of the machine are equivalent. Show your work. That is, produce one or more tables of the form

```
  1
  2
  3
  4
  5
  6
  7
  8
```

with increasingly many Xs marking pairs of states that are known to be non-equivalent and show those tables in your solution.

(b) Draw the state diagram for the equivalent **minimal** state DFA for $L(D)$.

3. (From D. Kozen) For any set $A$ of natural numbers, define

- **binary** $A = \{w \mid w \in \{0, 1\}^* \text{ and the number represented by } w \text{ in binary is an element of } A \}$
- **unary** $A = \{1^n \mid n \in A\}$

For example, if $A = \{4, 9, 16\}$, then

- **binary** $A = \{100, 1001, 10000\}$
- **unary** $A = \{1111, 11111111, 11111111111111\}$
Consider the following two propositions:

- For all \( A \), if \textbf{binary} \( A \) is regular, then so is \textbf{unary} \( A \).
- For all \( A \), if \textbf{unary} \( A \) is regular, then so is \textbf{binary} \( A \).

One of these statements is true and the other is false. Which is which? Justify your claims by either providing a proof or a counterexample.

Some hints:

- Think about closure properties. If you can describe a language as a union or intersection of a finite collection of languages that are clearly regular, then the language is regular.
- When thinking about DFAs for unary languages, consider what happens if you limit yourself to examples of DFAs that only contain a single final state.

4. Complete exercise 2.13 from Sipser (as modified below):

Let \( G = (V, \Sigma, R, S) \) be the following grammar. \( V = \{S, T, U\} \); \( \Sigma = \{0, \#\} \); and \( R \) is the set of rules:

\[
\begin{align*}
S & \rightarrow TT \mid U \\
T & \rightarrow 0T \mid T0 \mid \# \\
U & \rightarrow 0U00 \mid \#
\end{align*}
\]

a. Describe \( L(G) \) in English. (Note: A mix of English and extended regular expression notation may be more effective.)

b. Prove that \( L(G) \) is not regular using the Myhill-Nerode Theorem.