CS371 Spring 2007 Assigned: Fri. 2/9/07
Prof. Morgan McGuire Due: Wed. 2/14/07 10:00am

Homework 1

_____
Name

As with all homework in this class, work on this alone. You may use calculators or computers for number crunching when working on this homework and refer to the internet and textbooks, but please close your book when actually writing the derivations (i.e., don't copy the answers directly from the book!)

1. Derive an expression for the first intersection (i.e., the one closest to the ray origin) between a ray and the surface of a sphere. Let the ray have origin $\vec{x}$ and unit direction $\vec{\omega}$. Let the sphere have radius r and be centered at $\vec{c}$.

2. Derive an expression for the first intersection between a ray and a plane. Let the ray have origin $\vec{x}$ and unit direction $\vec{\omega}$. Let the plane contain the point $\vec{p}$ and have unit surface normal $\vec{n}$.

3. The incident radiation from the sun on the Earth's surface is about 1000 W/m^2 when the sun is directly overhead. For this question, assume that sunlight all comes from the same direction and that all photons have a wavelength of 600 nanometers. Ignore any atmospheric effects on the photons, assume that the earth is a perfect sphere, and assume that the Sun is small enough/far enough away to be approximated as a point when viewed from the Earth.

a) Approximately how many sunlight photons strike a $1m^2$ patch of the Earth's surface on the Equator between 12:00pm (noon) and 12:01pm on the day of the vernal equinox (which is when the Sun passes directly over the Equator)?
Recall $e_\lambda = h * c / \lambda$; $h = 6 \times 10^{-34}$ J s; $c = 3 \times 10^8$ m / s.

b) For the same patch in part (a), how much energy is incident in a 24-hour period? Assume that the Earth rotates perfectly on its North-South axis on the day in question, so that sunrise and sunset are exactly twelve hours apart.

c) An orange Volkswagen New Beetle has a mass of about 1400 kg. If you could create a perfect solar cell to capture the energy from part (b), how high could you lift the Volkswagen using this energy to power a perfect lift? Assume that the acceleration due to gravity is 10 m/s^2 directed downward.

4. Describe how you *would* derive the intersection a ray with origin $\vec{x}$ and unit direction $\vec{\omega}$ with a triangle whose vertices are $\vec{v}_0$, $\vec{v}_1$, and $\vec{v}_2$. You do not have to actually derive the expression!

5. Consider a scene containing an infinitely small white light source that radiates equally in all directions and a 1m^2 card in the shape of a planar disk. The card is at distance $r > 0$ from the light and oriented so that its normal always points directly towards the light from the center of the card. The total rate of energy emission by the light (i.e., power of the light) is 10 W. What is the total power directly incident on the card due to the light as a function of r? *Hints: this problem is set up so that you do not have to consider the angle of incidence inside the integral, like you did (or should have done!) in problem 3. Try solving the 2D version of this problem first if you have trouble, and then generalize to 3D.*