Introduction to Probability

Why Randomness

- **Randomization.** We allow a fair coin flip in unit time.
- Why randomize?
  - Deterministic algorithms offer little flexibility
  - Randomness often enables to surprisingly simple & fast algorithms
- Very important in computer science:
  - Symmetry-breaking protocols, memory management, learning algorithms, contention resolution, hashing, load balancing, cryptographic, AI, game theory
- Gives insight in “real world” issues
  - Polling, risk assessment, scientific testing, gambling, etc.

Probability Review

- Before we design/analyze randomized algorithms, we need a foundation in probability
- Plan: we'll start with some things you've likely seen before
  - Will be a “review" of probability from Discrete Math
  - Since each Math 200 differs, ensure everyone has same background
- Will move on to randomized algorithms and data structures:
  - Hashing
  - Skip lists
  - Fingerprinting
  - etc.

“Deathbed” Formulas

- You should remember these even on your deathbed [MAB]
- *Extremely* useful in probability
- $\left(1+\frac{1}{n}\right)^n \approx e \qquad \left(1-\frac{1}{n}\right)^n \approx \frac{1}{e}$ for large enough n (gets close quite quickly)
- More precisely: $\left(1+\frac{1}{n}\right)^n \leq e \qquad \left(1-\frac{1}{n}\right)^n \leq \frac{1}{e}$
- $\left(\frac{x}{y}\right)^y \leq \binom{x}{y} \leq \left(\frac{ex}{y}\right)^y$

$\binom{x}{y} = \frac{x!}{y!(x-y)!}$ is the number of y-sized subsets of x items

Acknowledgments

- Some of the material in these slides are taken from
  - Kleinberg Tardos Slides by Kevin Wayne (https://www.cs.princeton.edu/~wayne/kleinberg-tardos/pdf/04GreedyAlgorithmsI.pdf)
  - Jeff Erickson's Algorithms Book (http://jeffe.cs.illinois.edu/teaching/algorithms/book/Algorithms-JeffE.pdf)
  - Hamiltonian cycle reduction images from Michael Sipser's Theory of Computation Book