Divide and Conquer: Sorting and Recurrences
Divide & Conquer: The Pattern

- **Divide** the problem into several independent smaller instances of exactly the same problem
- **Delegate** each smaller instance to the *Recursive Leap of Faith* (technically known as induction hypothesis)
- **Combine** the solutions for the smaller instances
Review: Merge Sort

MergeSort($L$):

if $L$ has one element
    return $L$

Divide $L$ into two halves $A$ and $B$

$A \leftarrow \text{MergeSort}(A)$
$B \leftarrow \text{MergeSort}(B)$

$L \leftarrow \text{Merge}(A, B)$

return $L$
Merge Step: $\Theta(n)$

- Scan sorted lists from left to right
- Compare element by element; create new merged list

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`i` and `j` are indices pointing to the current elements being compared.
Merge Step: $\Theta(n)$

Is $a[i] \leq b[j]$ ?
- Yes, $a[i]$ appended to $c$, advance $i$
- No, $b[j]$ appended to $c$, advance $j$

merged list $c$
Merge Step: $\Theta(n)$

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![Diagram showing the merge step of two sorted lists](image)
Yada yada yada...
Merge Step: $\Theta(n)$

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merged list $c$
Acknowledgments

• Some of the material in these slides are taken from
  • Jeff Erickson’s Algorithms Book (http://jeffe.cs.illinois.edu/teaching/algorithms/book/Algorithms-JeffE.pdf)