Stable Matching Problem :: Definitions

Problem statement. Given a set H of n hospitals, a set D of n doctors, and their respective preferences, can we construct a *perfect matching* of doctors to hospitals that is *stable*—that is, no doctor-hospital pair has an incentive to break their match.

Perfect Matching: A matching M is perfect if it assigns each doctor to a single hospital and each hospital to a single doctor.

Stable matching: A matching M is *stable* if there is no pair $(d, h) \in H \times D$ where both:

1. d prefers h to its current match in M, and
2. h prefers d to its current match in M.

Propose-Reject Algorithm by Gale Shapley

Let's examine the Gale-Shapley propose-reject algorithm:

Algorithm 1 Propose-Reject - finds a stable matching

Initialize each doctor d and hospital h as Free
while there is a free doctor who hasn't proposed to every hospital **do**
    Choose a free doctor d
    $h \leftarrow$ first hospital on d's list to whom d has not yet proposed
    if h is Free **then**
        d and h are Matched
    else if h prefers d to its current match d' **then**
        d and h are Matched and d' is Free
    else
        h rejects d and d remains Free
    end if
end while