Lecture 25: Project Euler Day 2

The Fibonacci sequence is defined by the recurrence relation:

$$F_n = F_{n-1} + F_{n-2},$$

where $F_1 = 1$ and $F_2 = 1$.

It turns out that F_{541}, which contains 113 digits, is the first Fibonacci number for which the last nine digits are 1-9 pandigital (contain all the digits 1 to 9, but not necessarily in order). And F_{2749}, which contains 575 digits, is the first Fibonacci number for which the first nine digits are 1-9 pandigital.

Given that F_k is the first Fibonacci number for which the first nine digits AND the last nine digits are 1-9 pandigital, find k.

Working from left-to-right if no digit is exceeded by the digit to its left it is called an increasing number; for example, 134468.

Similarly if no digit is exceeded by the digit to its right it is called a decreasing number; for example, 66420.

We shall call a positive integer that is neither increasing nor decreasing a “bouncy” number; for example, 155349.

Clearly there cannot be any bouncy numbers below one-hundred, but just over half of the numbers below one-thousand (525) are bouncy. In fact, the least number for which the proportion of bouncy numbers first reaches 50% is 538.

Surprisingly, bouncy numbers become more and more common and by the time we reach 21780 the proportion of bouncy numbers is equal to 90%.

Find the least number for which the proportion of bouncy numbers is exactly 99%.

A number chain is created by continuously adding the square of the digits in a number to form a new number until it has been seen before.
For example,

$$44 \to 32 \to 13 \to 10 \to 1 \to 1$$

$$85 \to 89 \to 145 \to 42 \to 20 \to 4 \to 16 \to 37 \to 58 \to 89$$

Therefore any chain that arrives at 1 or 89 will become stuck in an endless loop. What is most amazing is that EVERY starting number will eventually arrive at 1 or 89.

How many starting numbers below ten million will arrive at 89?

Some positive integers n have the property that the sum$(n + \text{reverse}(n))$ consists entirely of odd (decimal) digits. For instance,

$$36 + 63 = 99 \text{ and } 409 + 904 = 1313.$$

We will call such numbers reversible; so 36, 63, 409, and 904 are reversible. Leading zeroes are not allowed in either n or reverse(n).

There are 120 reversible numbers below one-thousand.

How many reversible numbers are there below one-billion (10^9)?