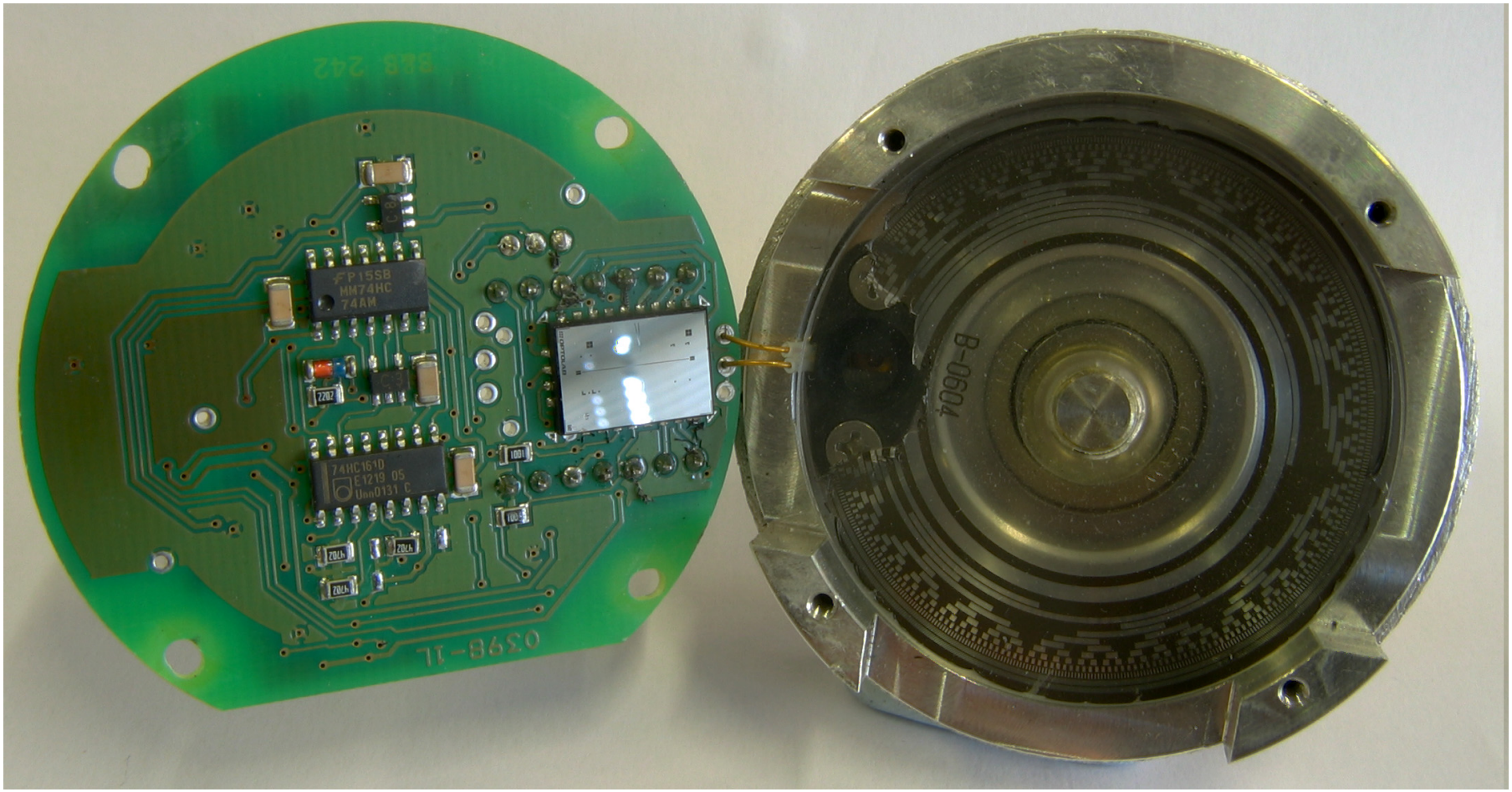
Lecture 11: Gray Codes

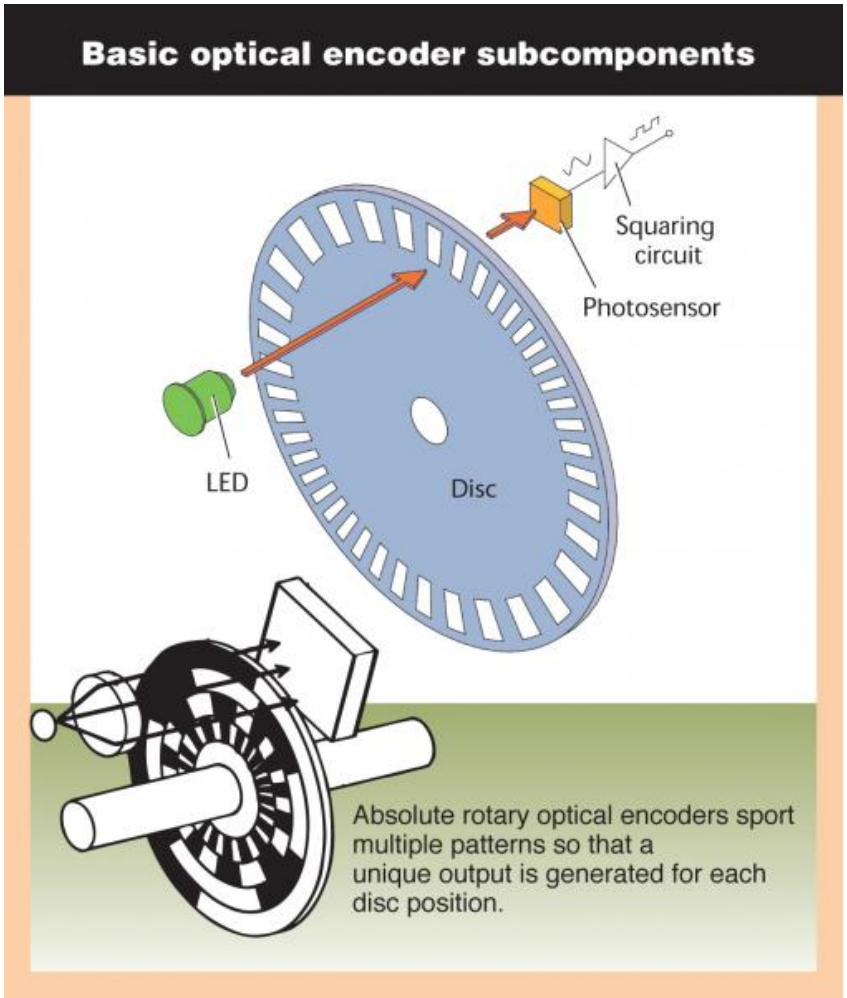
(Re)Orienting Ourselves

Binary encodings can be used to track position in a rotary encoder.



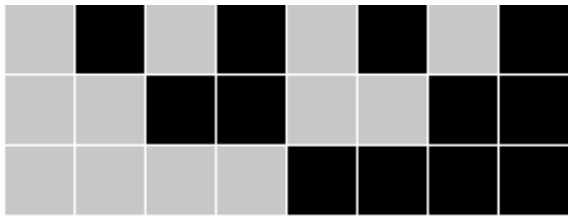
(Re)Orienting Ourselves

Binary optimal rotary encoder.



(Re)Orienting Ourselves - Binary Numbers

- Each "track" has an associated sensor
  - "Off" corresponds to a '0', "On" to a '1'
- The combined state of all sensors tells us our position



(Re)Orienting Ourselves - Binary Numbers

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  - "Off" corresponds to a '0', "On" to a '1'
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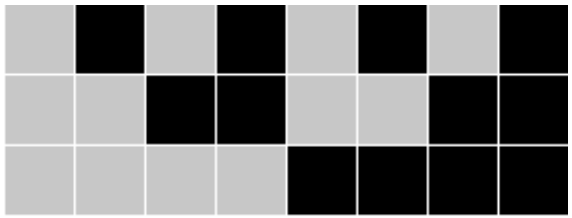
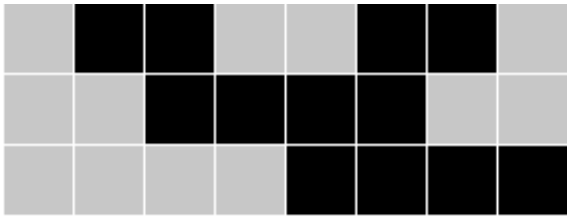


- Consider the changes between states '2' and '1' vs. states '2' and '3'
  - Multiple bits change at a time.
  - What if our sensors are not perfectly aligned?

(Re)Orienting Ourselves - Gray Codes

A Gray code is any numerical code where consecutive integers are represented by binary numbers that differ in exactly one digit.



Sensor alignment does not matter for changes between adjacent states!

(note the properties are preserved when the sequence wraps around)

Binary Reflected Gray Codes

We can build an $(n+1)$-bit Gray code from an n-bit Gray code:

1. Copy the sequence (creating an 'original' and a 'copy')
2. Reverse the order of the elements in the 'copy' sequence (hence the name binary-*reflected* Gray code)
3. Prefix each element in the 'original' sequence with a '0'
4. Prefix each element in the reversed 'copy' with a '1'
5. Concatenate the 'original' sequence and the 'copy' sequence

The $n = 1$ Gray code is $0, 1$.

Binary Reflected Gray Codes

Initial Sequence	**Copy the Sequence**	**Reflect the copy**	**'0' + original '1' + copy**
0	0	0	00
1	1	1	01
	0	1	11
	1	0	10

Binary Reflected Gray Codes

Initial Sequence	**Copy the Sequence**	**Reflect the copy**	**'0' + original '1' + copy**
00	00	00	000
01	01	01	001
11	11	11	011
10	10	10	010
	00	10	110
	01	11	111
	11	01	101
	10	00	100

Binary Reflected Gray Codes

Initial Sequence	**Copy the Sequence**	**Reflect the copy**	**'0' + original '1' + copy**
000	000	000	0000
001	001	001	0001
011	011	011	0011
010	010	110	0010
110	110	110	0110
111	111	111	0111
101	101	101	0101
100	100	100	0100
	000	100	1100
	001	101	1101
	011	111	1111
	010	110	1110
	110	010	1010
	111	011	1011
	101	001	1001
	100	000	1000