CS134 Lecture 32:
Sorting

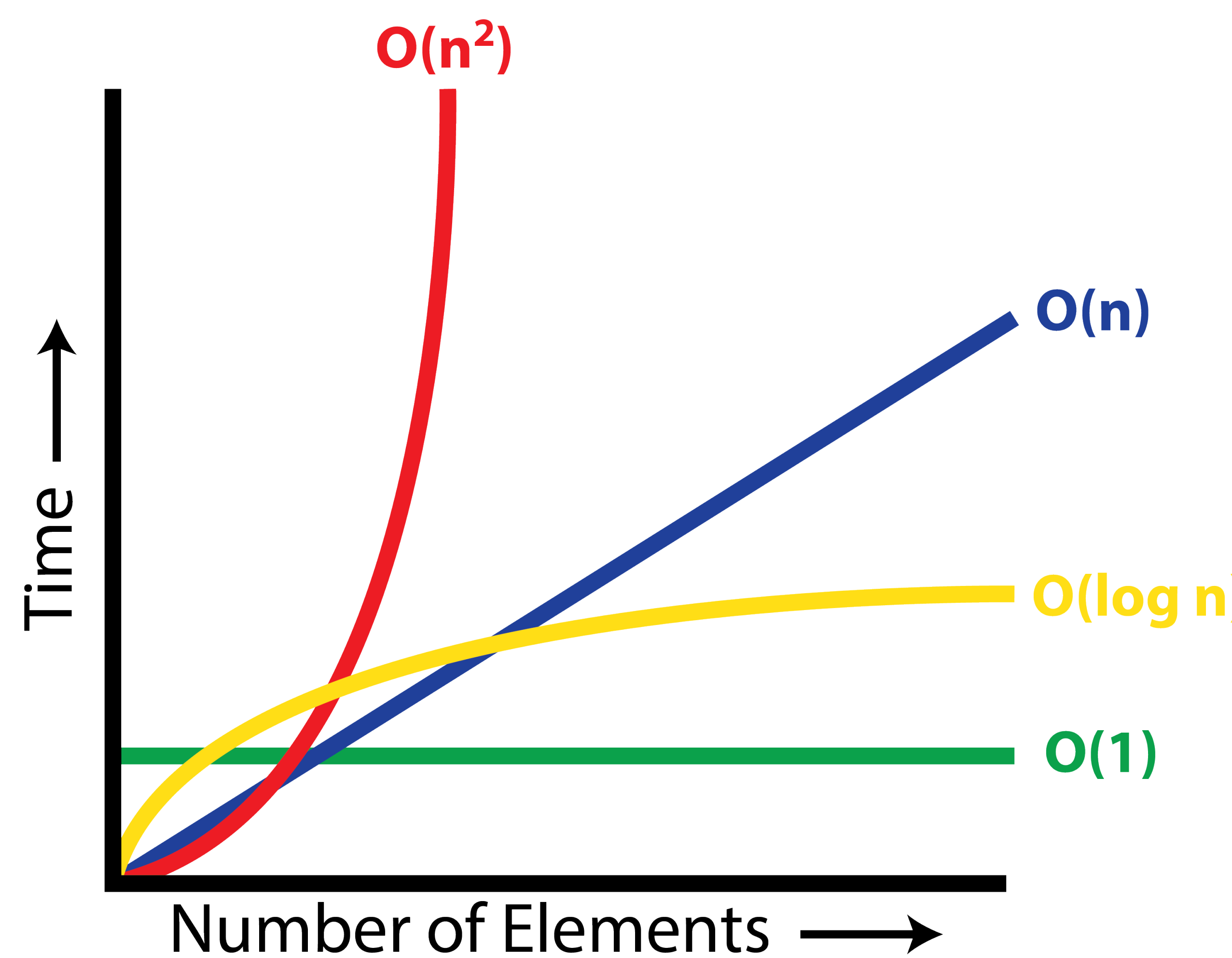
Announcements & Logistics

- **HW 10** will be released today, due Mon @ 10 pm
    - Last HW on efficiency and Big Oh
- **Lab 8** graded feedback will be returned soon
- **Lab 10** will be released today
    - Very short lab on searching and sorting (today's lecture)
    - No prelab
    - Individual lab but can discuss strategies with lab mate
- CS134 Scheduled Final: **Friday, May 17, 9:30 AM**
    - Room: **TCL 123**

Do You Have Any Questions?

Last Time: Efficiency & Searching

- Measured efficiency as number of steps taken by algorithm on worst-case inputs of a given size
- Introduced Big-O notation: captures the rate at which the number of steps taken by the algorithm grows wrt size of input n, "as n gets large"
- Compared array lists vs linked lists
- Compared linear vs binary search



Today: Searching and Sorting

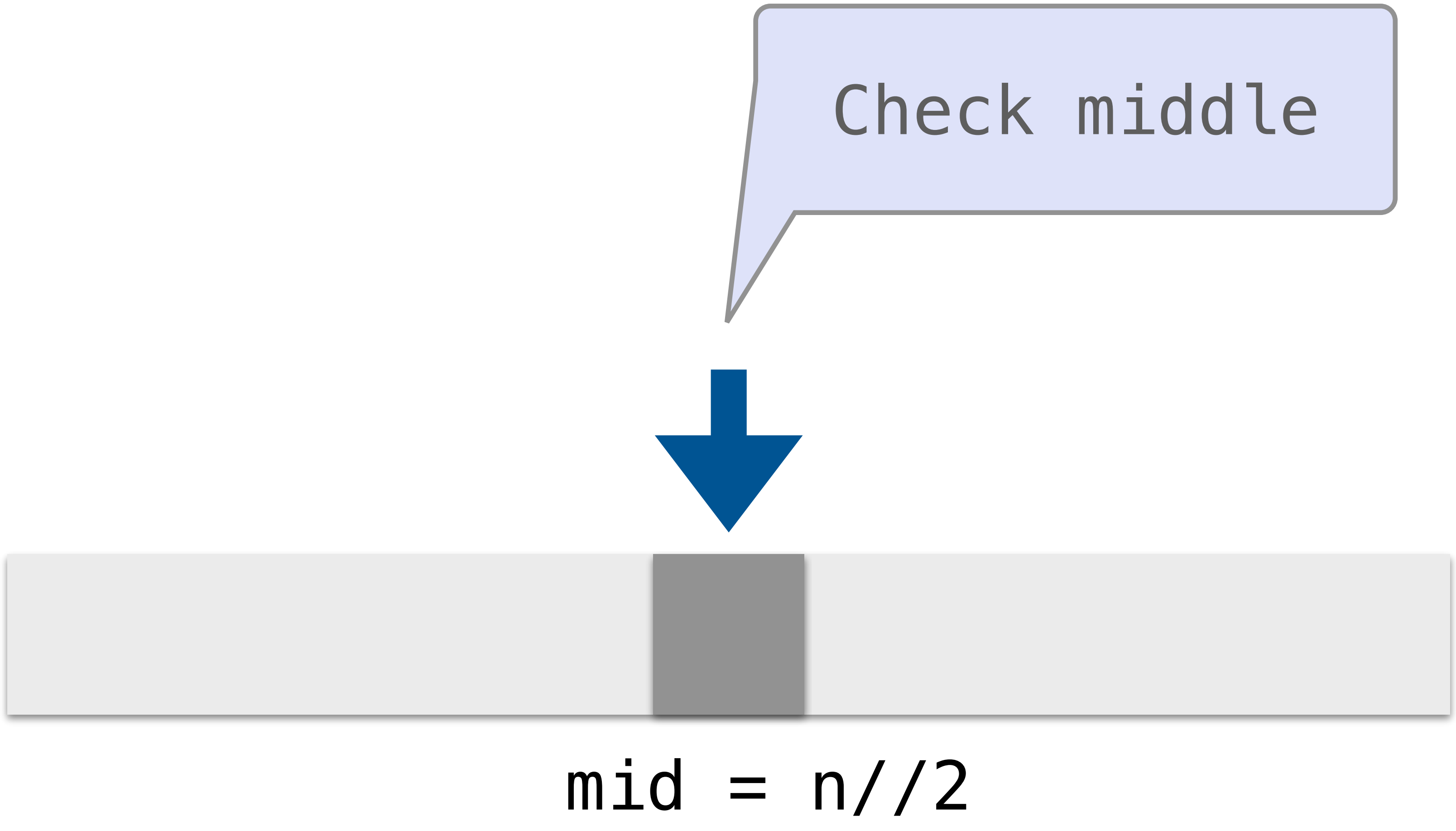
- Review recursive implementation of binary search
- Discuss some classic sorting algorithms:
    - ***Selection sort***
    - ***Merge sort***

Binary Search

- The **recursive search algorithm** we described to search in a sorted array is called **binary search**
- It can be much more efficient than a **linear search**
  - Takes $\approx \log n$ lookups if we can index into sequence efficiently
- Which data structure supports fast access/indexing?
  - Accessing an item at index i in an array requires constant time
  - Accessing an item at index i in a LinkedList can require traversing the whole list (even if it is sorted!): linear time
- To get a more efficient search algorithm, it is not only important to use the right algorithm, we need to use the right data structure as well!

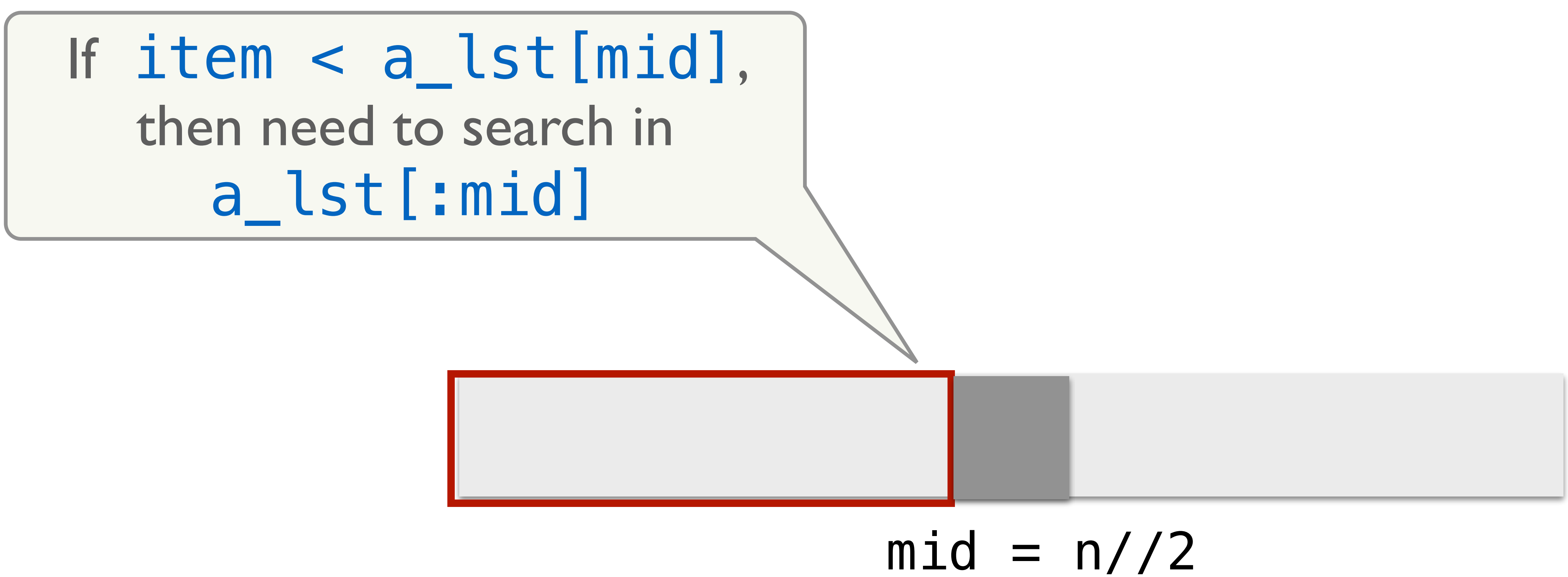
Binary Search

- Base cases? When are we done?
  - If list is too small (or empty) to continue searching, return False
  - If item we're searching for is the middle element, return True



Binary Search

- Recursive case:
    - Recurse on left side if item is smaller than middle
    - Recurse on right side if item is larger than middle



Binary Search

- Recursive case:
  - Recurse on left side if item is smaller than middle
  - Recurse on right side if item is larger than middle

```python
def binary_search(seq, item):
    """Assume seq is sorted. If item is
    in seq, return True; else return False."""

    n = len(seq)

    # base case 1
    if n == 0:
        return False

    mid = n // 2
    mid_elem = seq[mid]

    # base case 2
    if item == mid_elem:
        return True

    # recurse on left
    elif item < mid_elem:
        left = seq[:mid]
        return binary_search(left, item)

    # recurse on right
    else:
        right = seq[mid+1:]
        return binary_search(right, item)
```

Technically, there is one *small* problem with our implementation. List splicing is actually O(n)!

Binary Search: Improved

```
def binary_search_helper(seq, item, start, end):
    '''Recursive helper function used in binary search'''

    # base case 1
    if start > end:
        return False

    mid = (start + end) // 2
    mid_elem = seq[mid]

    if item == mid_elem:
        return True

    # recurse on left
    elif item < mid_elem:
        return binary_search_helper(seq, item, start, mid-1)

    # recurse on right
    else:
        return binary_search_helper(seq, item, mid+1, end)
```

Passing start/end indices as arguments avoids the need to splice!

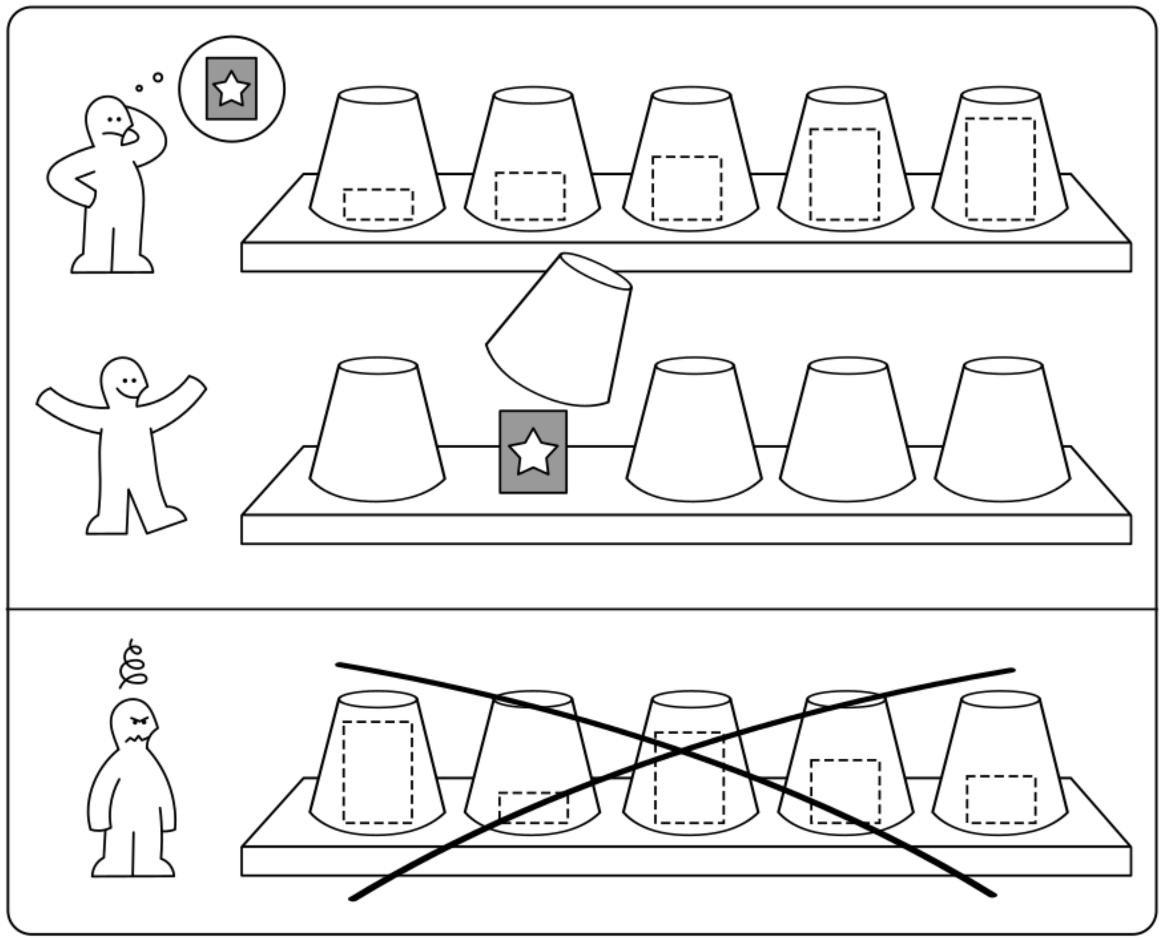
```
def binary_search_improved(seq, item):

    return binary_search_helper(seq, item, 0, len(seq)-1)
```

BINÄRY SEARCH

idea-instructions.com/binary-search/

IDEA



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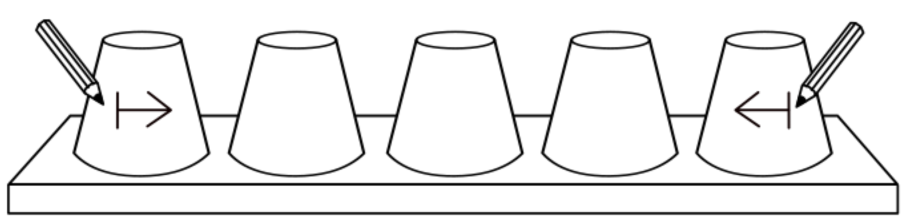

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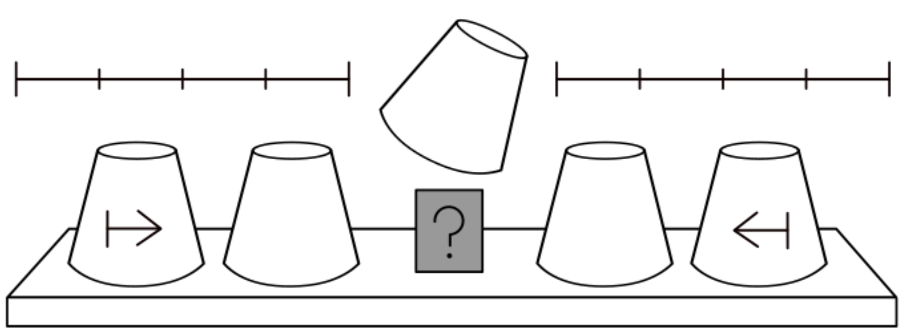

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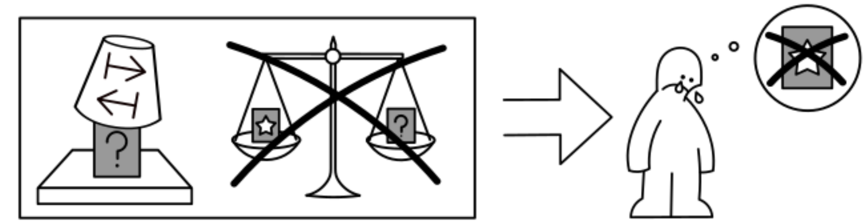

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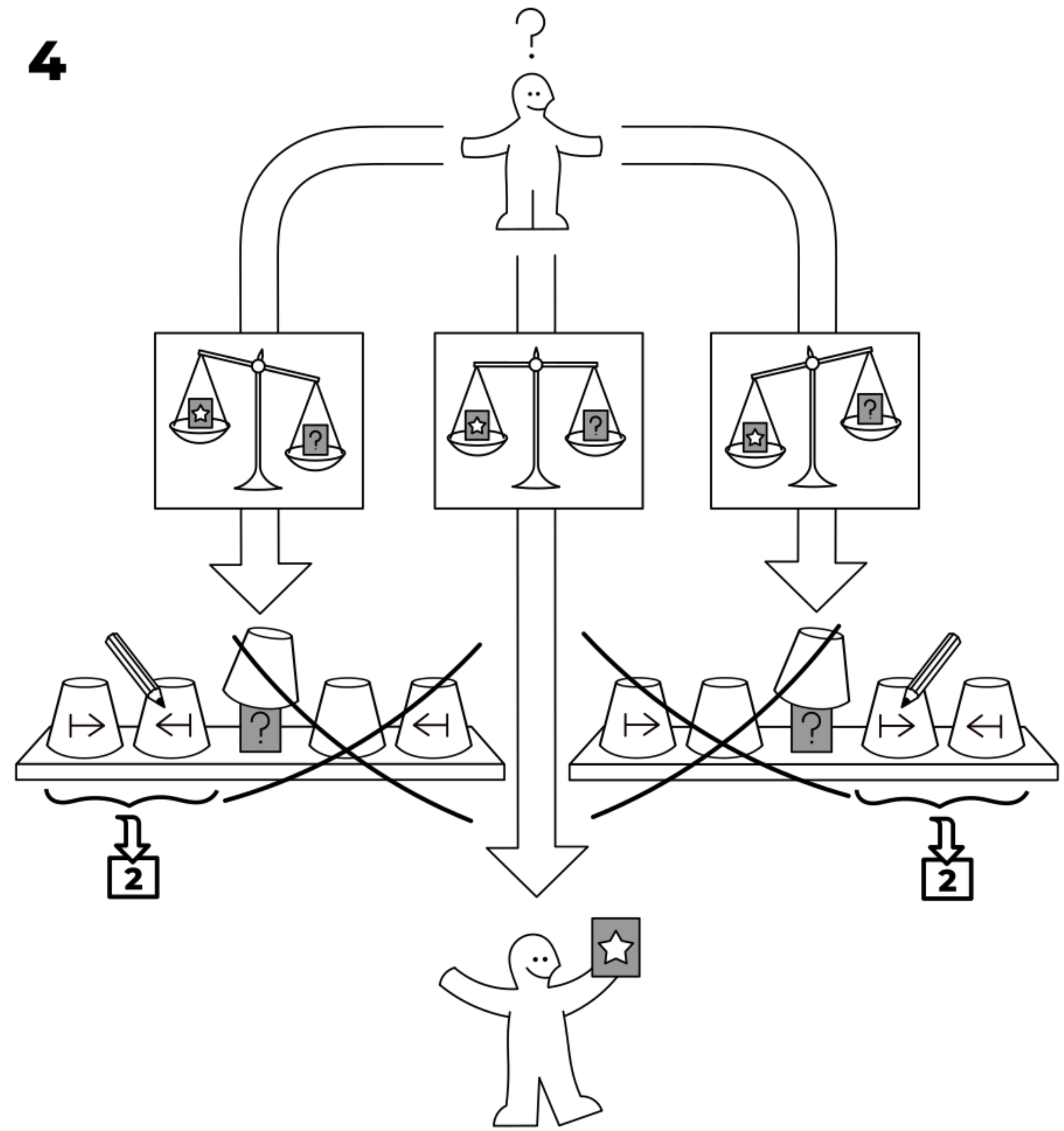


More on Big Oh

Big-O Notation

- Tells you how fast an algorithm is / the run-time of algorithms
  - But not in seconds!
- Tells you how fast the algorithm grows in number of operations

O(log n)

"Big O" Number of Operations

Understanding Big-O

- Notation: n often denotes the number of elements (size)
- **Constant time** or $O(1)$: when an operation does not depend on the number of elements, e.g.
  - Addition/subtraction/multiplication of two values, or defining a variable etc are all constant time
- **Linear time** or $O(n)$: when an operation requires time proportional to the number of elements, e.g.:

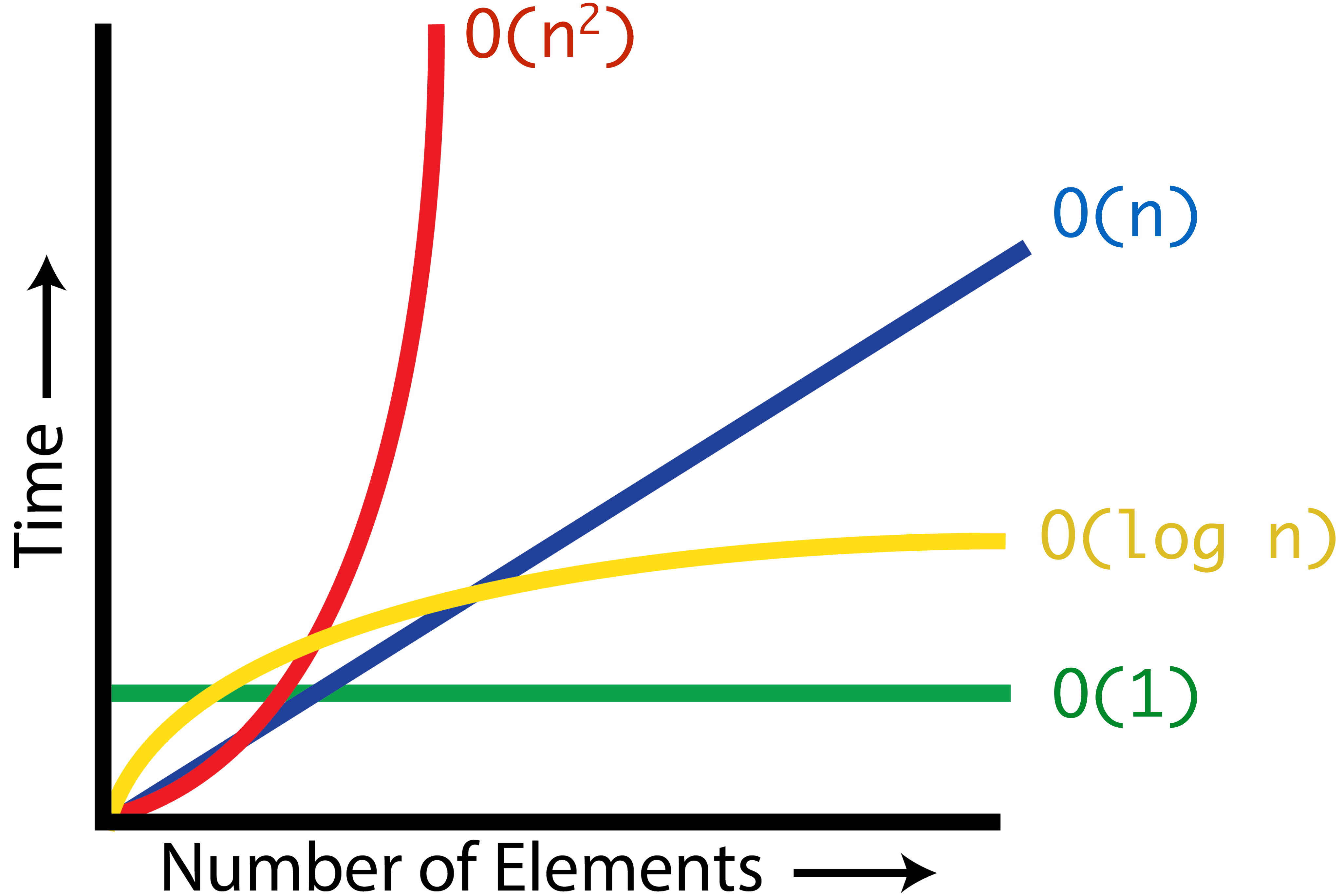
```
for item in seq:
    <do something>
```

- **Quadratic time** or $O(n^2)$: nested loops are often quadratic, e.g.,

```
for i in range(n):
    for j in range(n):
        <do something>
```

Big-O: Common Functions

- Notation: n often denotes the number of elements (size)
- Our goal: understand efficiency of some algorithms at a high level



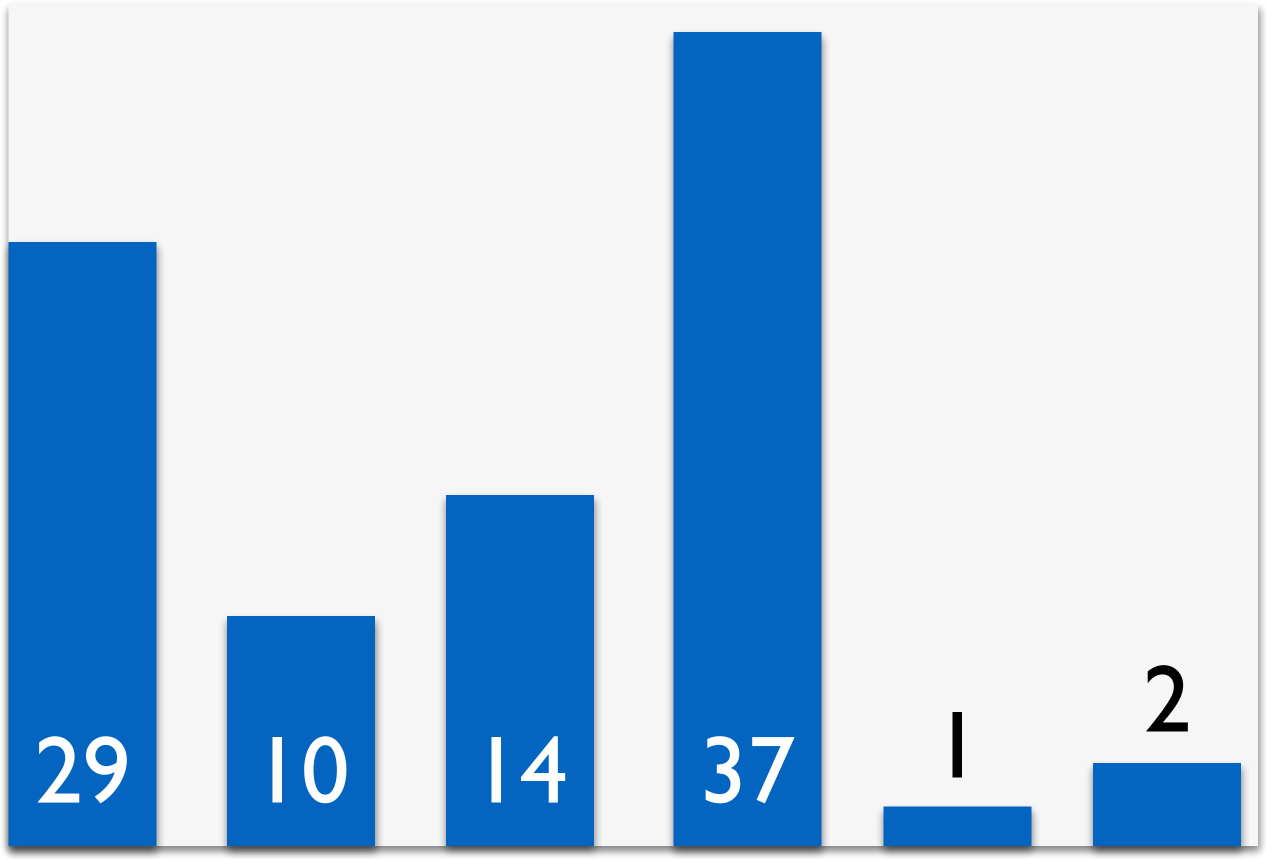
Sorting

Sorting

- **Problem:** Given a sequence of unordered elements, we need to sort the elements in ascending order.
- There are many ways to solve this problem!
- Built-in sorting functions/methods in Python
    - `sorted()`: ***function*** that returns a new sorted list
    - `sort()`: ***list method*** that mutates and sorts the list
- **Today:** how do we design our own sorting algorithm?
- **Question:** What is the best (most efficient) way to sort n items?
- We will use Big-O to find out!

Selection Sort

- A possible approach to sorting elements in a list/array:
  - Find the smallest element and move (swap) it to the first position
  - *Repeat*: find the second-smallest element and move it to the second position, and so on



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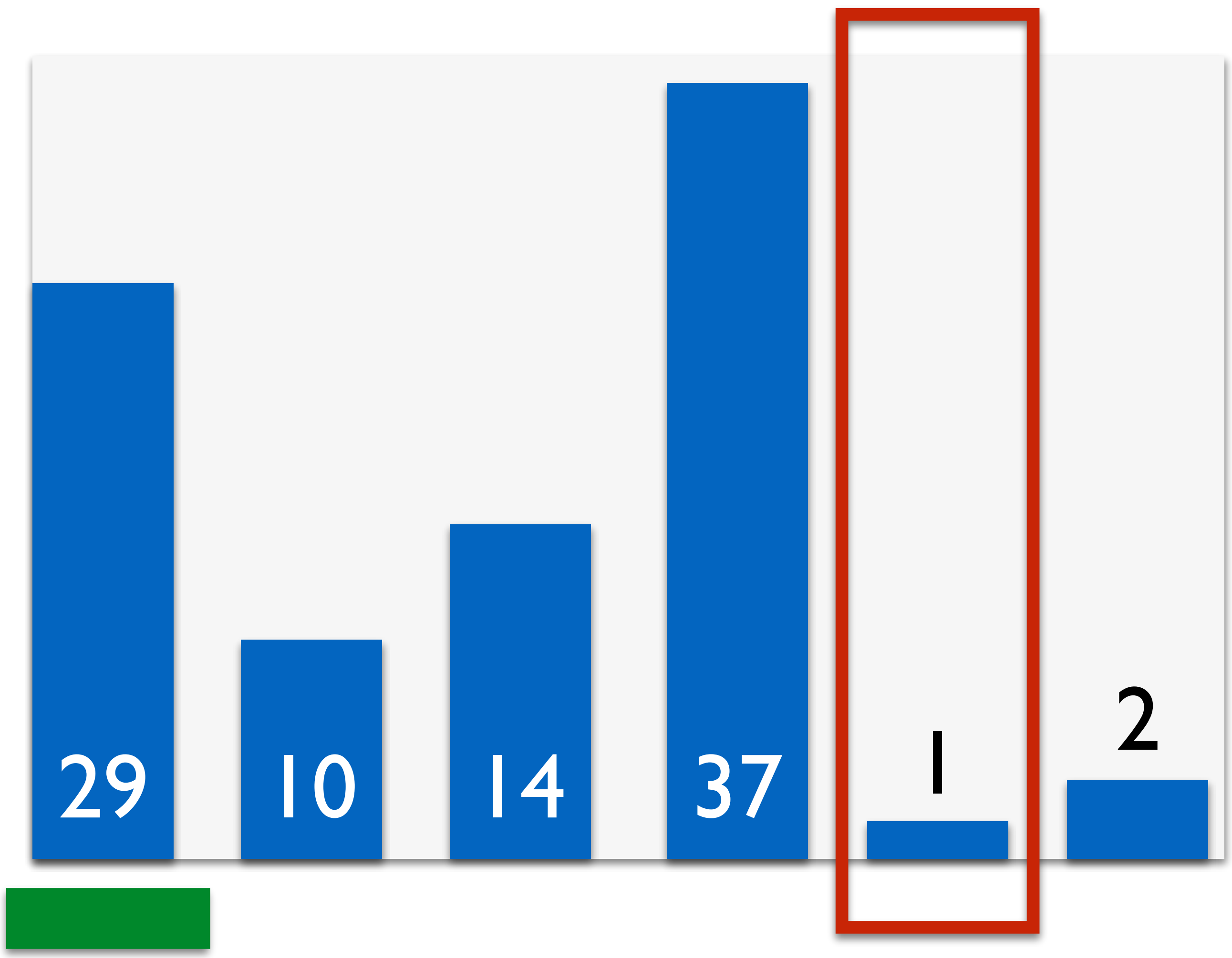

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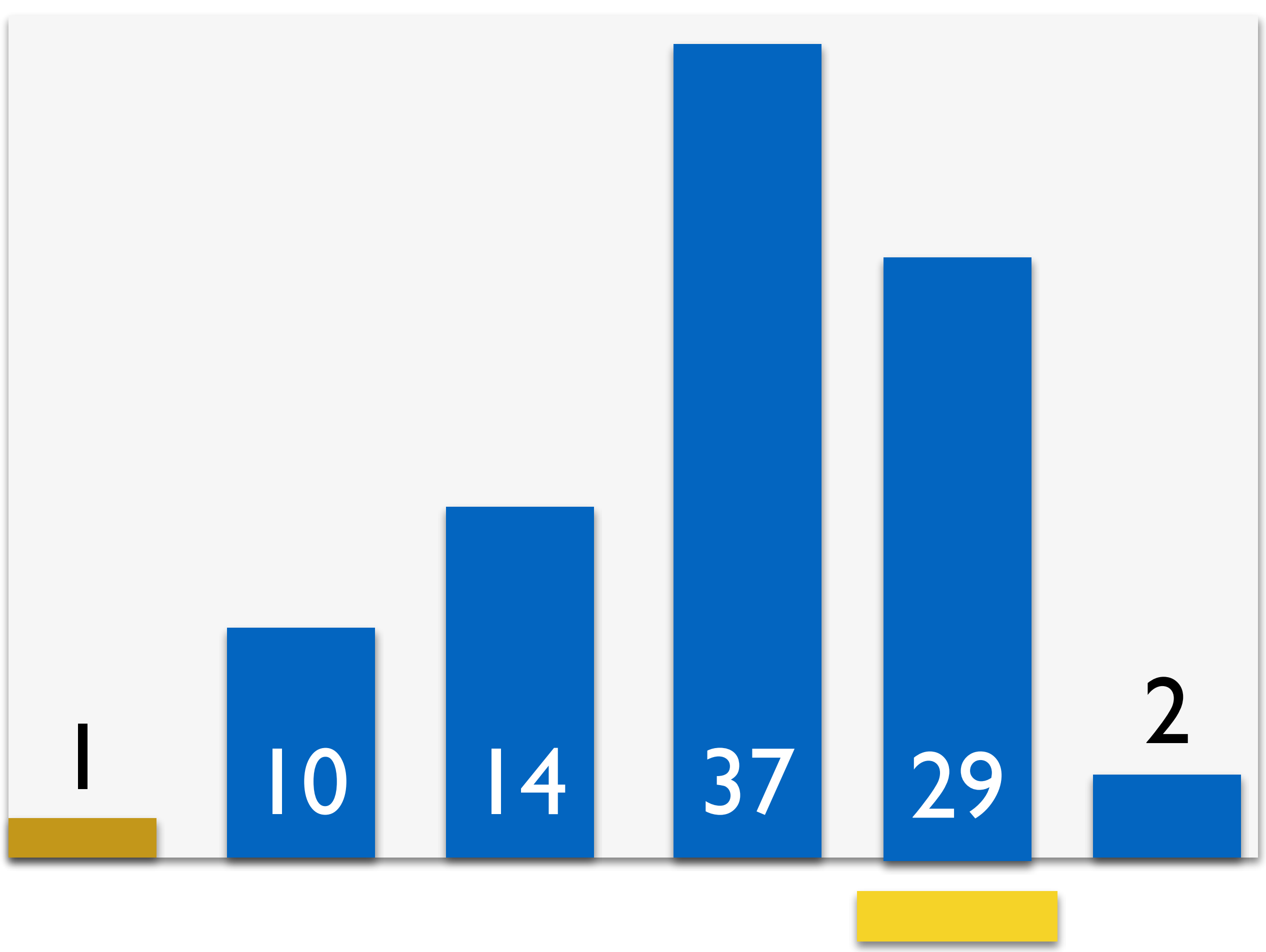

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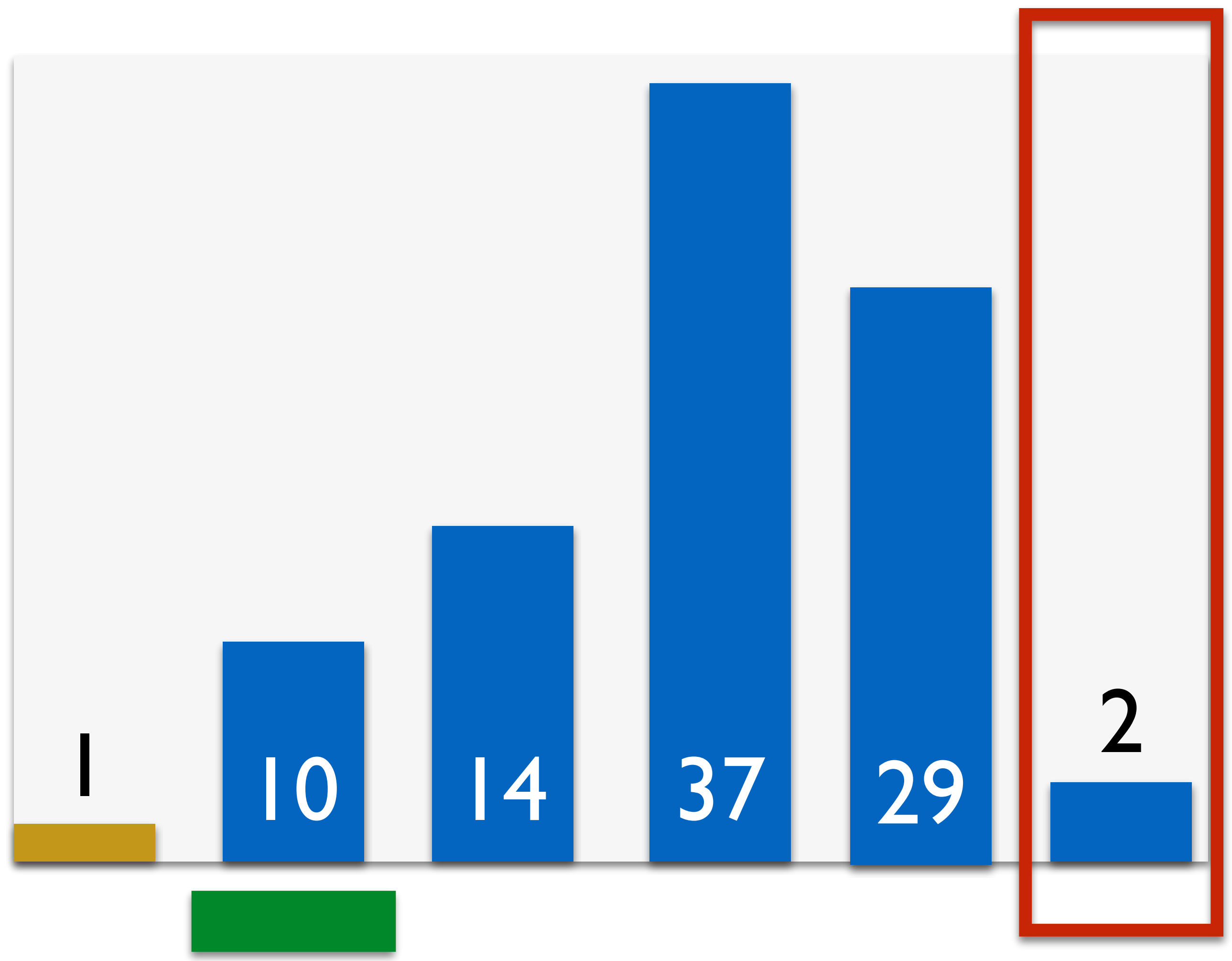
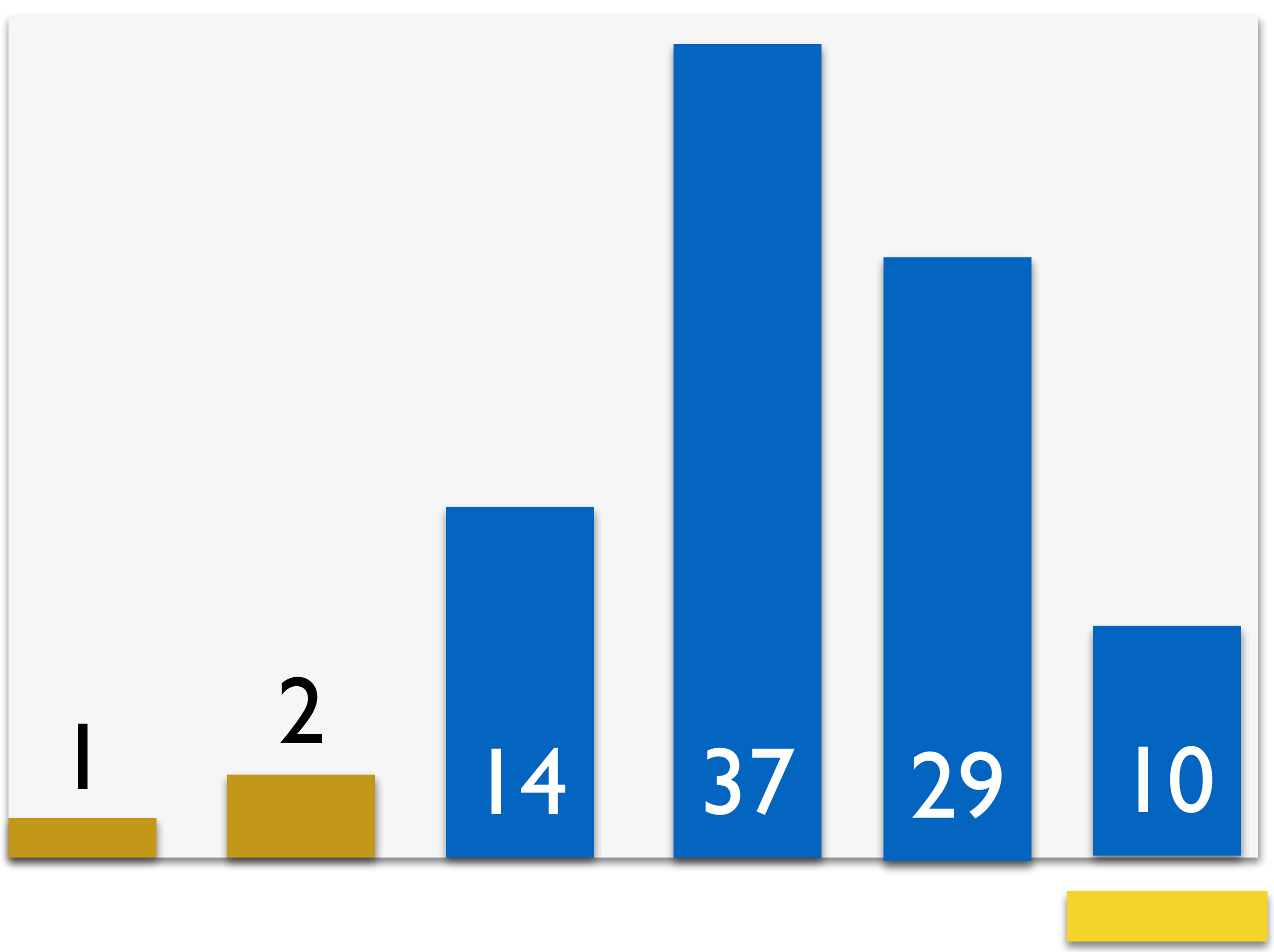


Selection Sort

- Find the smallest element and move (swap) it to the first position
- *Repeat*: find the second-smallest element and move it to the second position, and so on
- The gold bars represent the sorted portion of the list.



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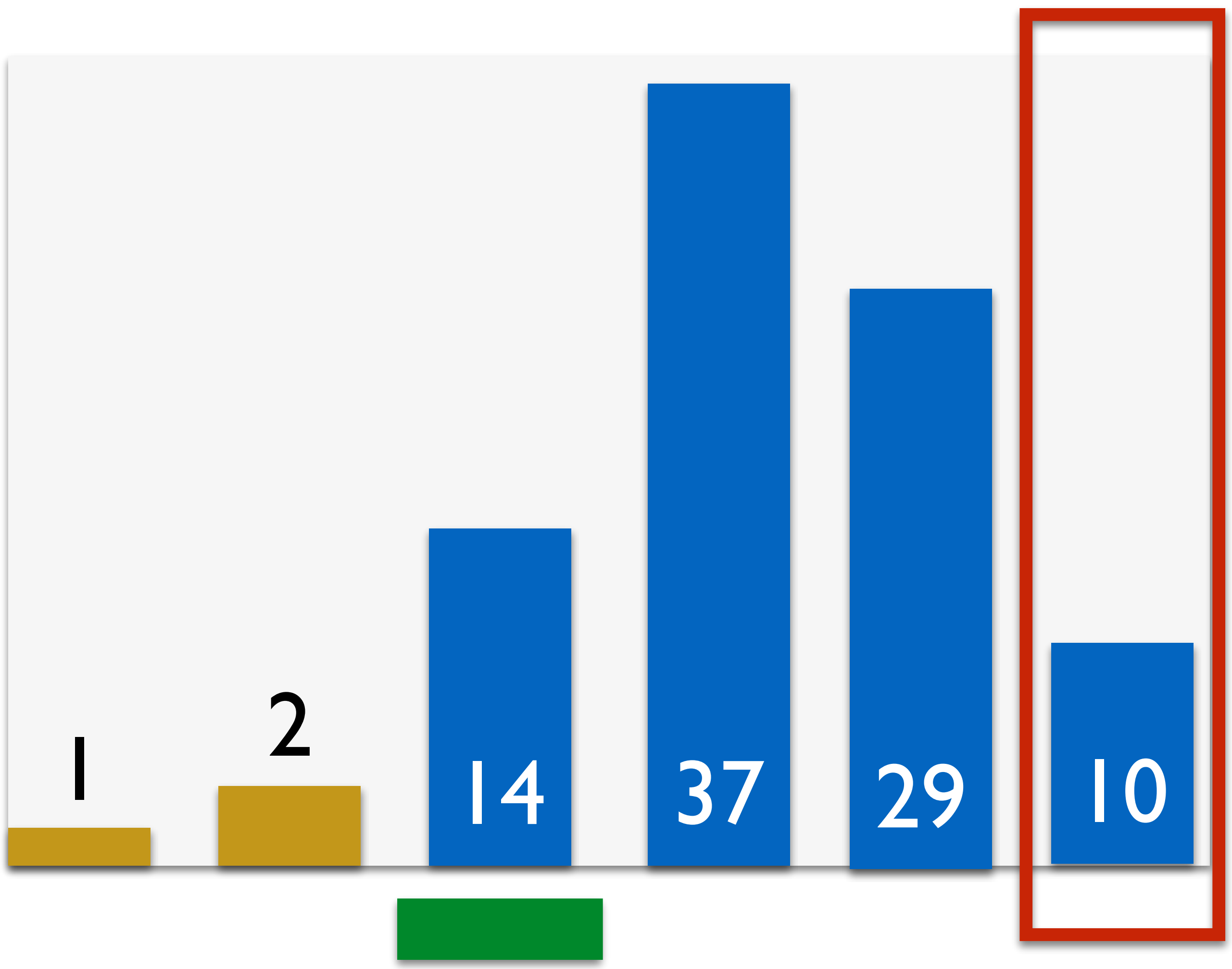

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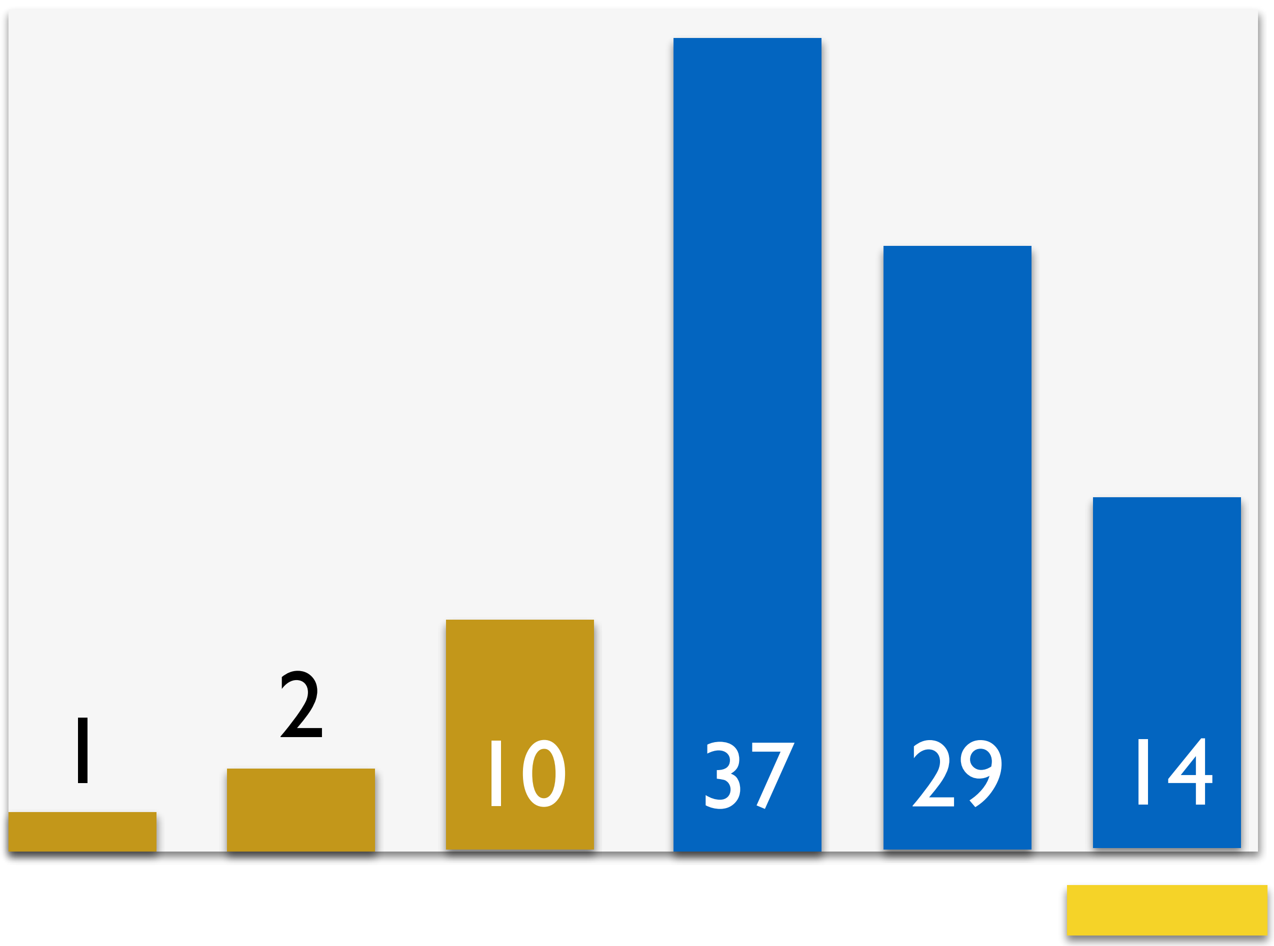

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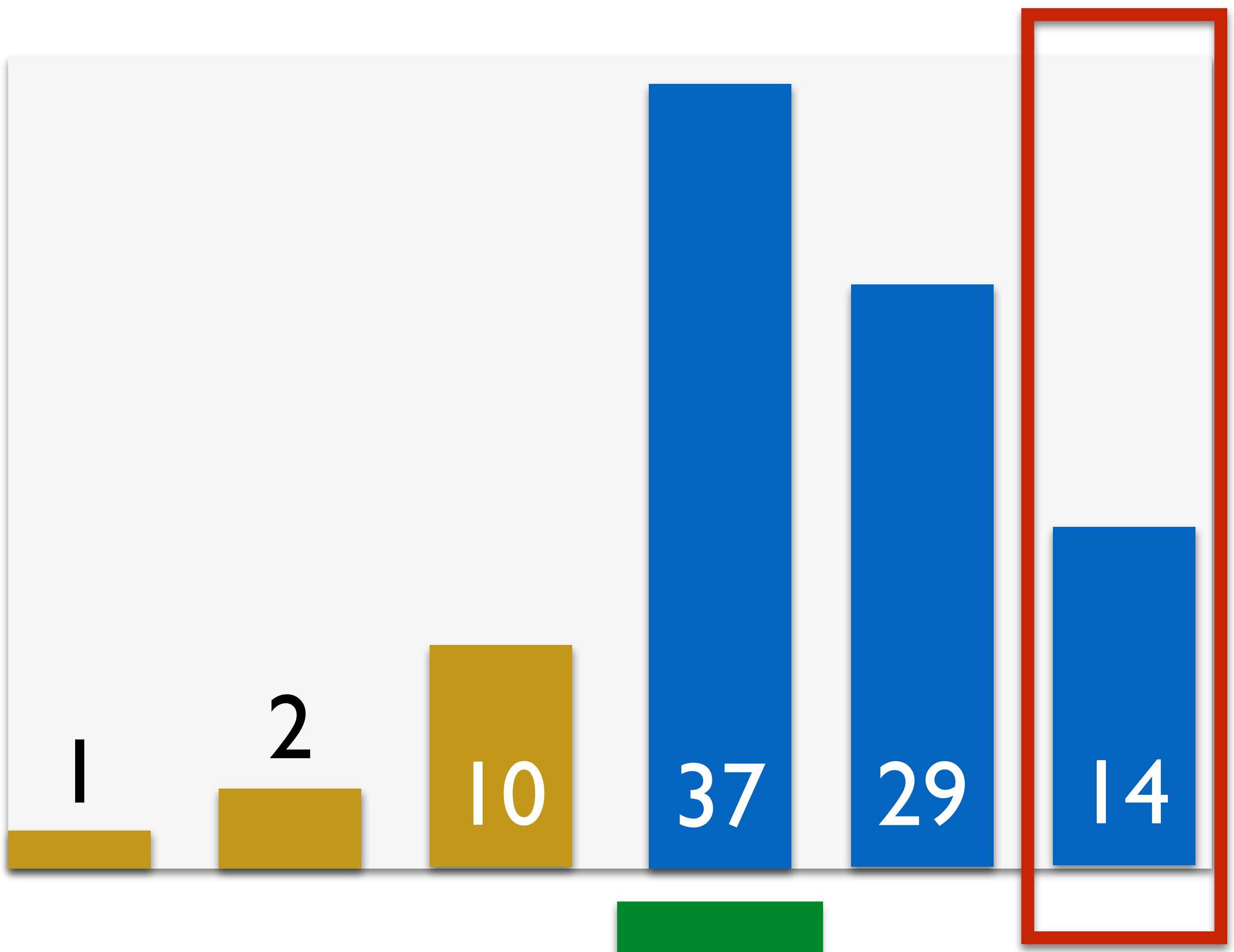

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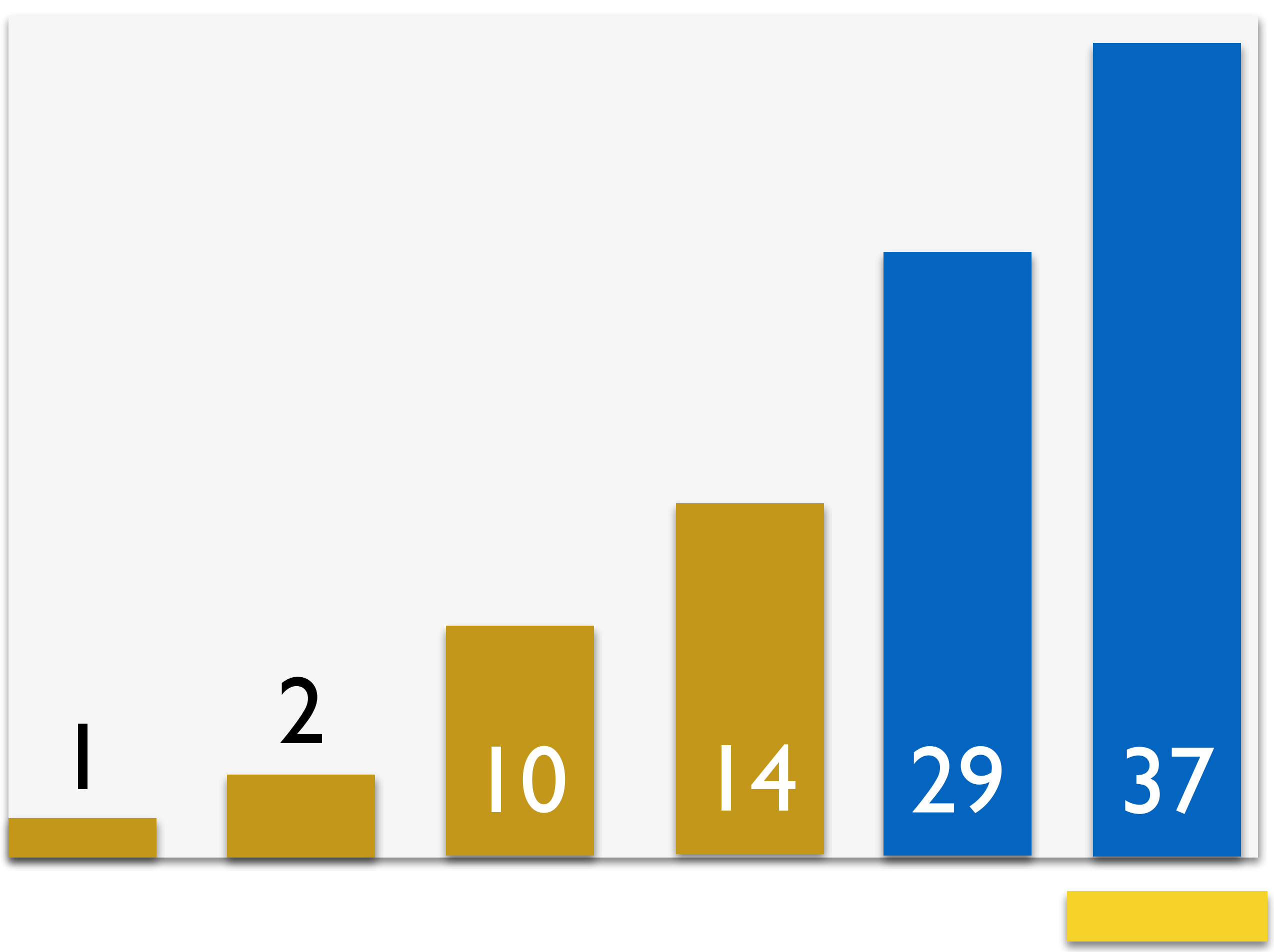

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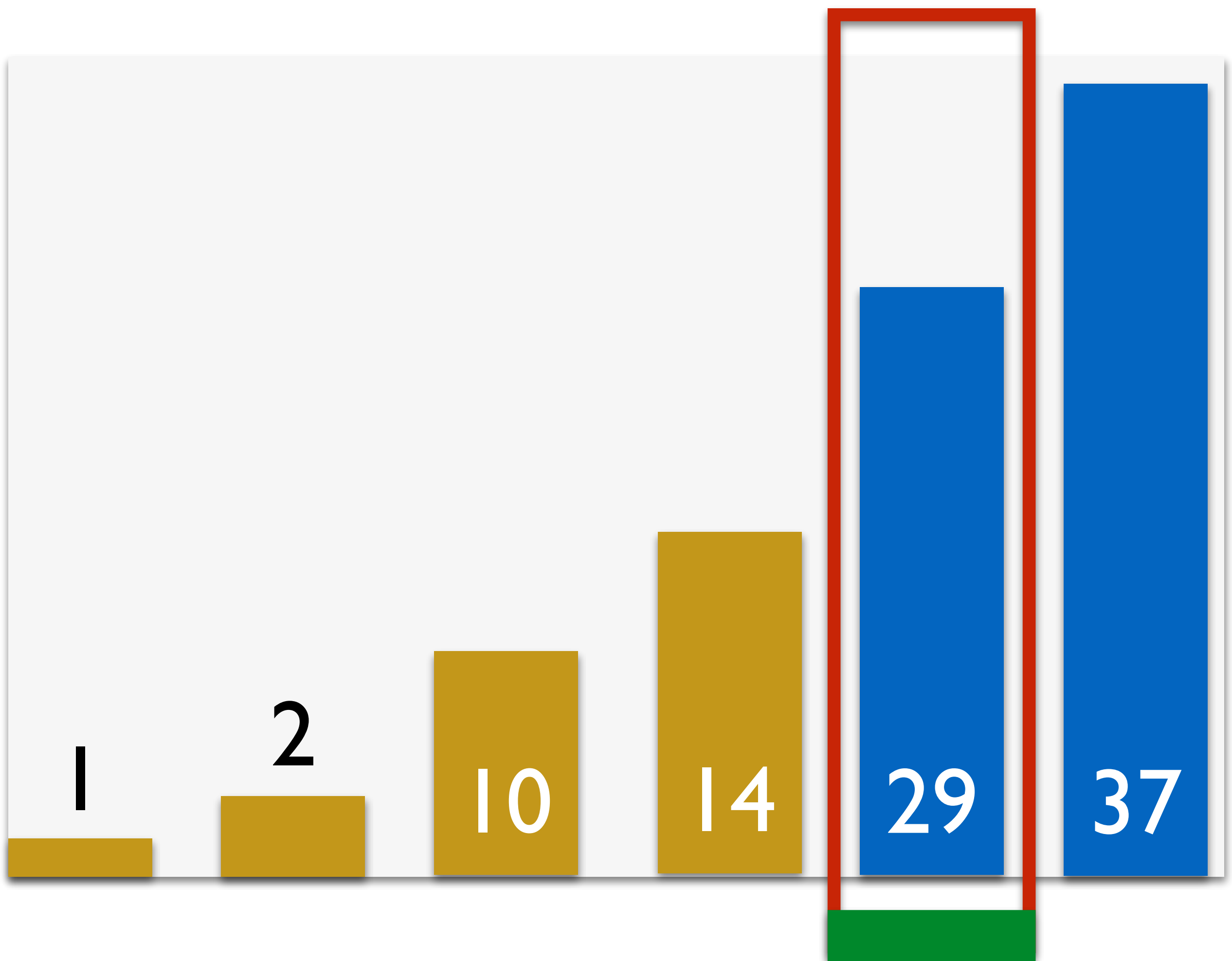

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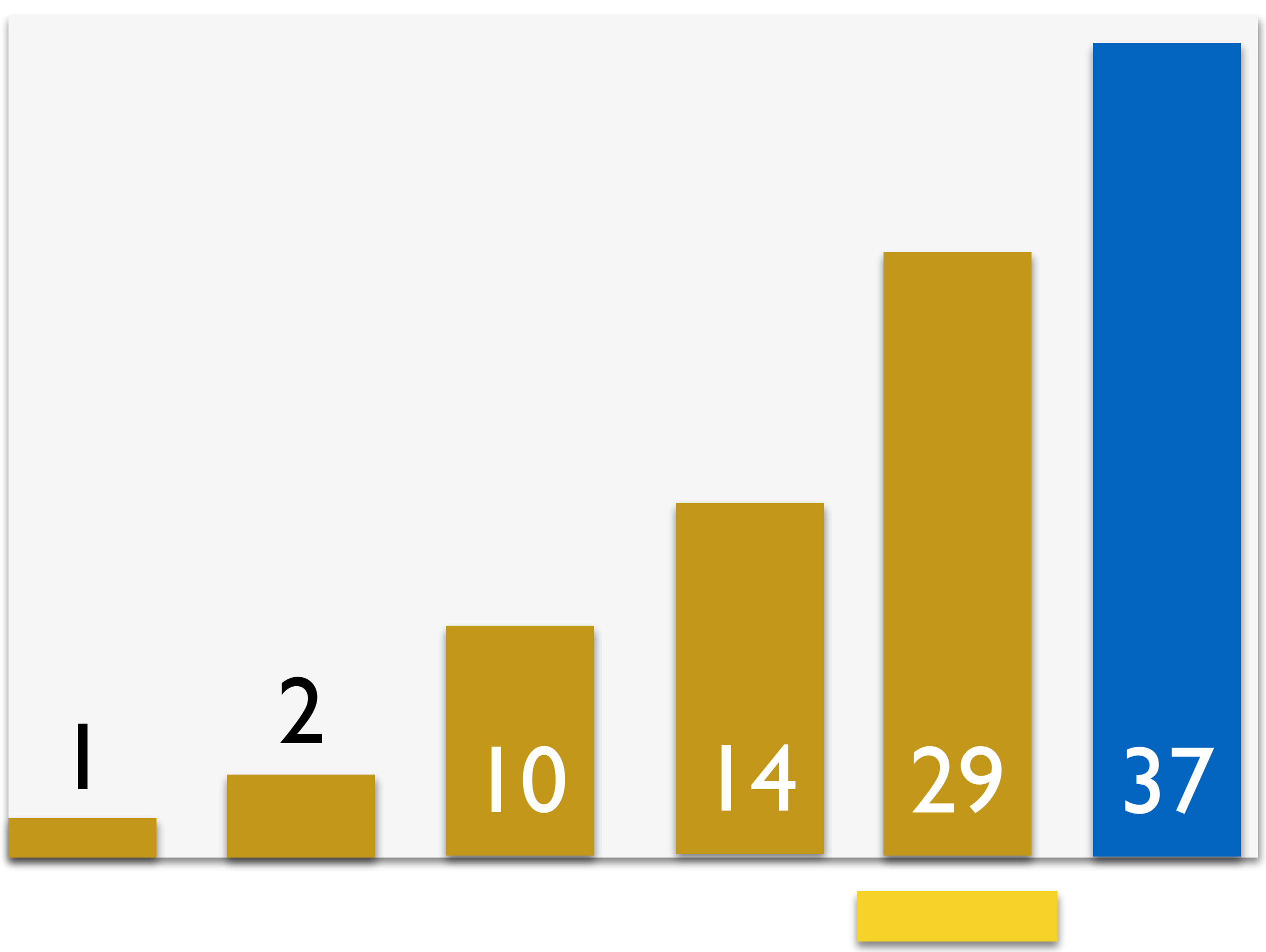

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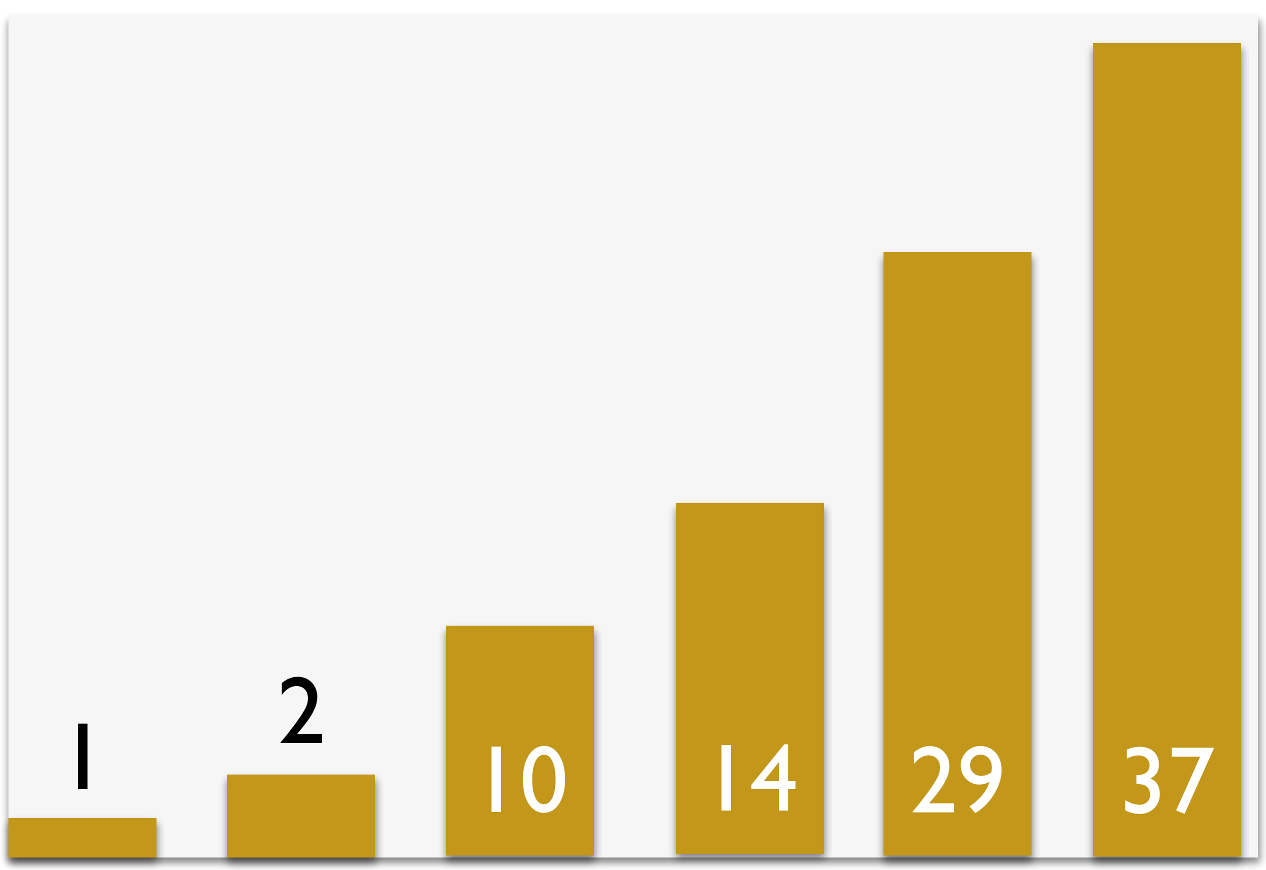

And now we're finally done!

Selection Sort

- Generalize: For each index i in the list `lst`, we need to find the **min** item in `lst[i:]` so we can replace `lst[i]` with that item
- In fact we need to find the position `min_index` of the item that is the minimum in `lst[i:]`
- **Reminder:** how to swap values of variables `a` and `b`?
  - in-line swapping: `a, b = b, a`
- How do we implement this algorithm?

Selection Sort

```
def selection_sort(my_lst):
    """Selection sort of a given mutable sequence my_lst,
    sorts my_lst by mutating it.  Uses selection sort."""

    # find size
    n = len(my_lst)

    # traverse through all elements
    for i in range(n):

        # find min element in the sublist from index i+1 to end

        min_index = get_min_index(my_lst, i)

        # swap min element with current element at i
        my_lst[i], my_lst[min_index] = my_lst[min_index], my_lst[i]
```

You will work on this helper function in Lab 10

Selection Sort

```
def selection_sort(my_lst):
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        # swap min element with current element at i
        my_lst[i], my_lst[min_index] = my_lst[min_index], my_lst[i]
```

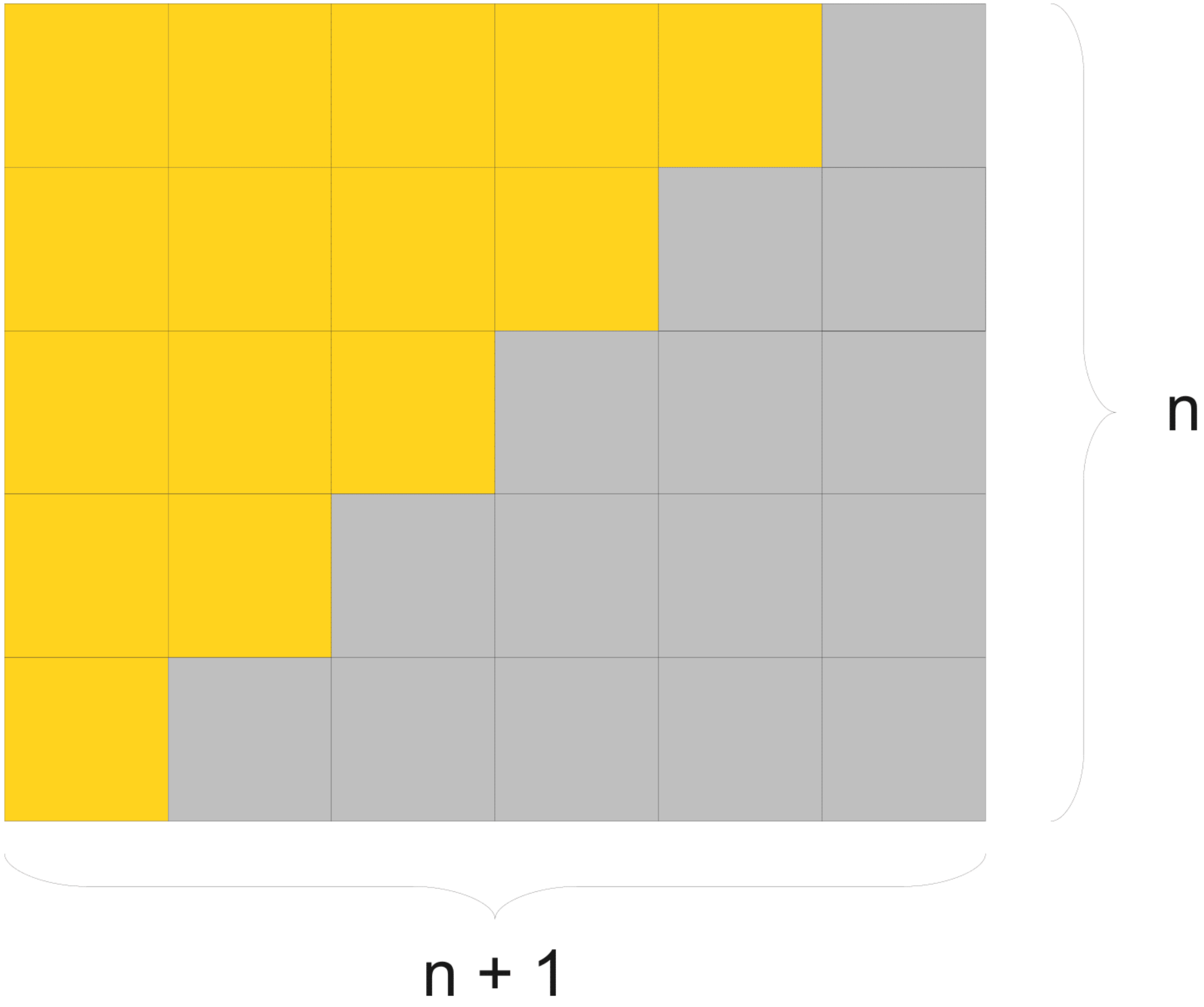
Even without an implementation, can we guess how many steps does this function need to take?

Selection Sort Analysis

- The helper function `get_min_index` must iterate through index `i` to `n` to find the min item
  - When `i = 0` this is `n` steps
  - When `i = 1` this is `n-1` steps
  - When `i = 2` this is `n-2` steps
  - And so on, until `i = n-1` this is `1` step
- Thus overall number of steps is sum of inner loop steps
$$(n-1) + (n-2) + \cdots + 0 \le n + (n-1) + (n-2) + \cdots + 1$$
- What is this sum? (You will see this in MATH 200 if you take it.)

Selection Sort Analysis: Visual

$$n + (n-1) + \ldots + 2 + 1 = n(n+1) / 2$$



Selection Sort Analysis: Algebraic

$$
\begin{array}{rl}
 & S = n + (n-1) + (n-2) + \cdots + 2 + 1 \\
+ & S = 1 + 2 + \cdots + (n-2) + (n-1) + n \\
\hline
\end{array}
$$

$$2S = (n+1) + (n+1) + \cdots + (n+1) + (n+1) + (n+1)$$

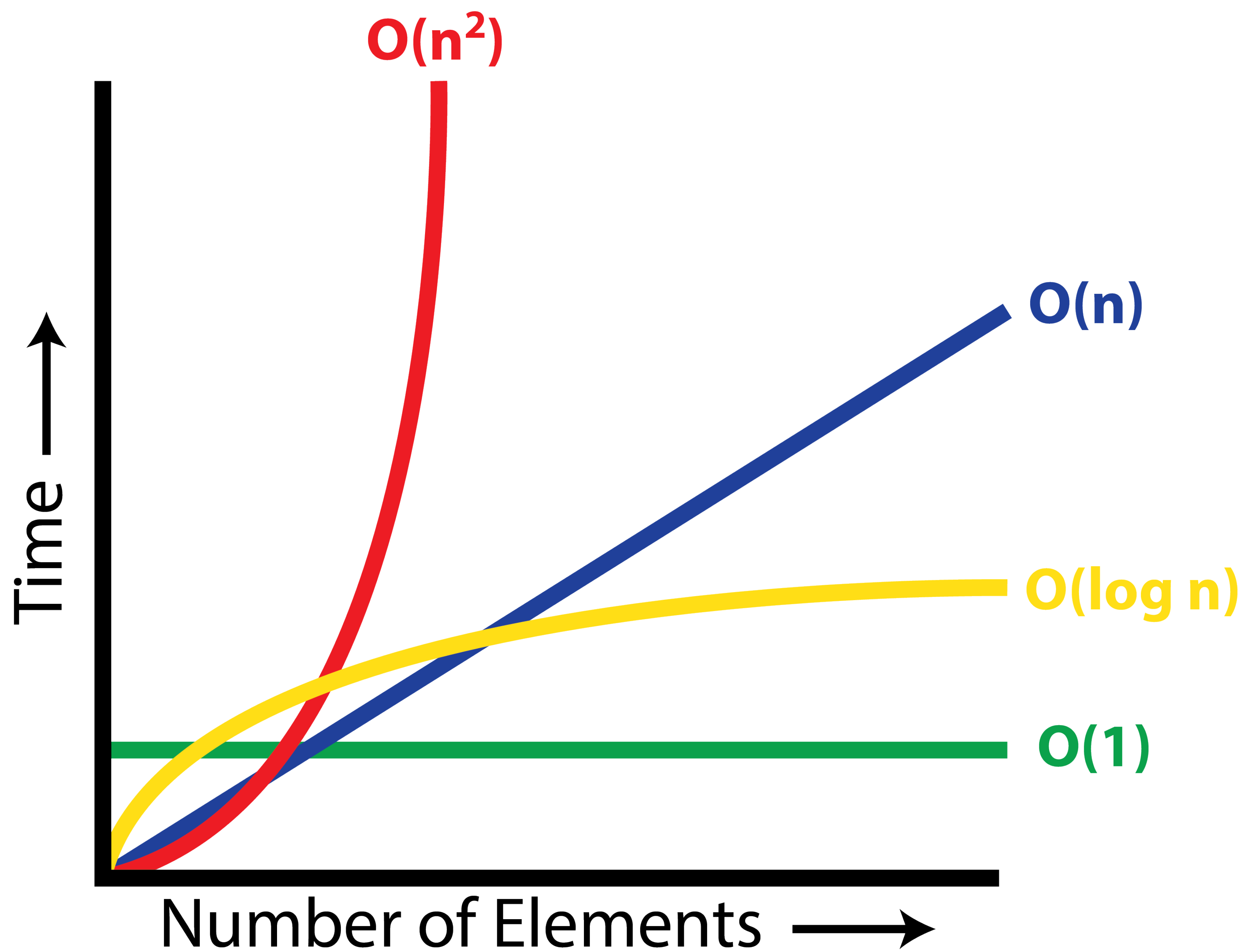
$$2S = (n+1) \cdot n$$

$$S = (n+1) \cdot n \cdot 1/2$$

- Total number of steps taken by selection sort is thus:
  - $O(n(n+1)/2) \ = O(n(n+1)) \ = O(n^2+n) \ = O(n^2)$

How Fast Is Selection Sort?

- Selection sort takes approximately n^2 steps!



More Efficient Sorting:

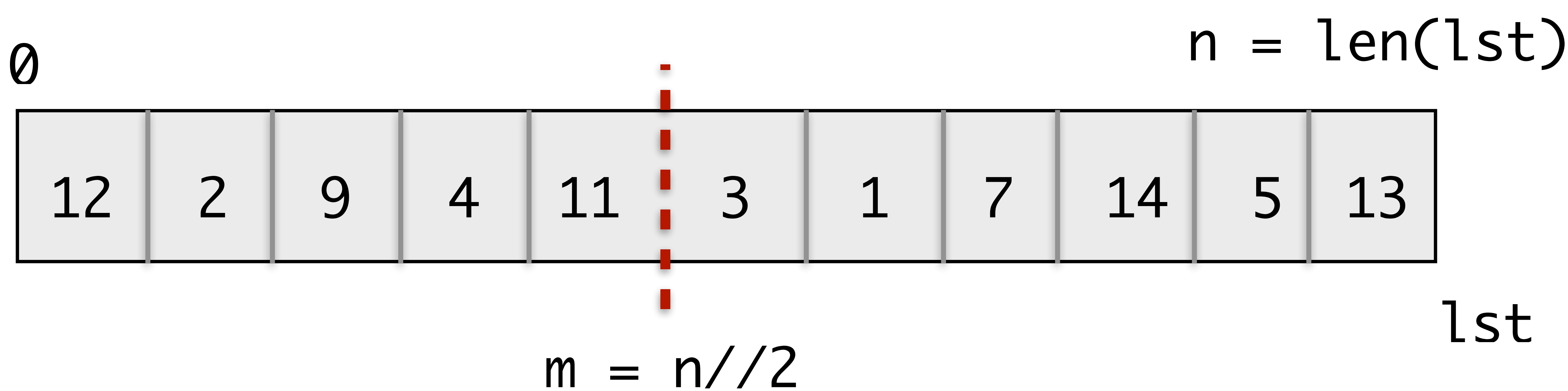
Merge Sort

Towards an $O(n \log n)$ Algorithm

- There are other sorting algorithms that compare and rearrange elements in different ways, but are still $O(n^2)$ steps
  - Any algorithm that takes n steps to move each item n positions (in the worst case) will take at least $O(n^2)$ steps
  - To do better than n^2, we need to move an item in fewer than n steps
- We can sort in $O(n \log n)$ time if we are clever: ***Merge sort algorithm*** (Invented by John von Neumann in 1945)

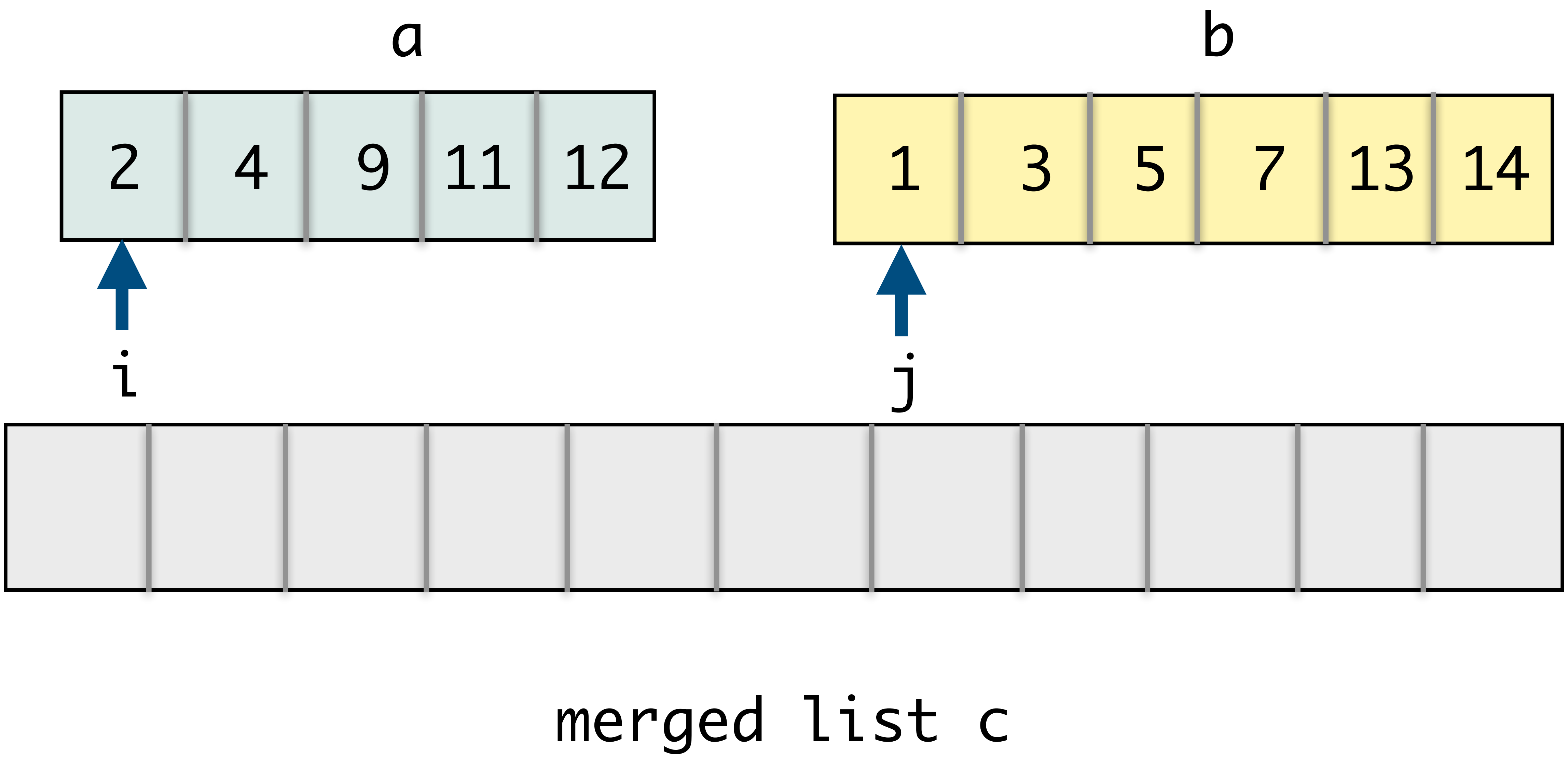
Merge Sort: Basic Idea

- If we split the list in half, sorting the left and right half are smaller versions of the same problem
- **Algorithm:**
  - **(Divide)** Recursively sort left and right half ($O(\log n)$)
  - **(Unite)** Merge the sorted halves into a single sorted list ($O(n)$)



Merging Sorted Lists

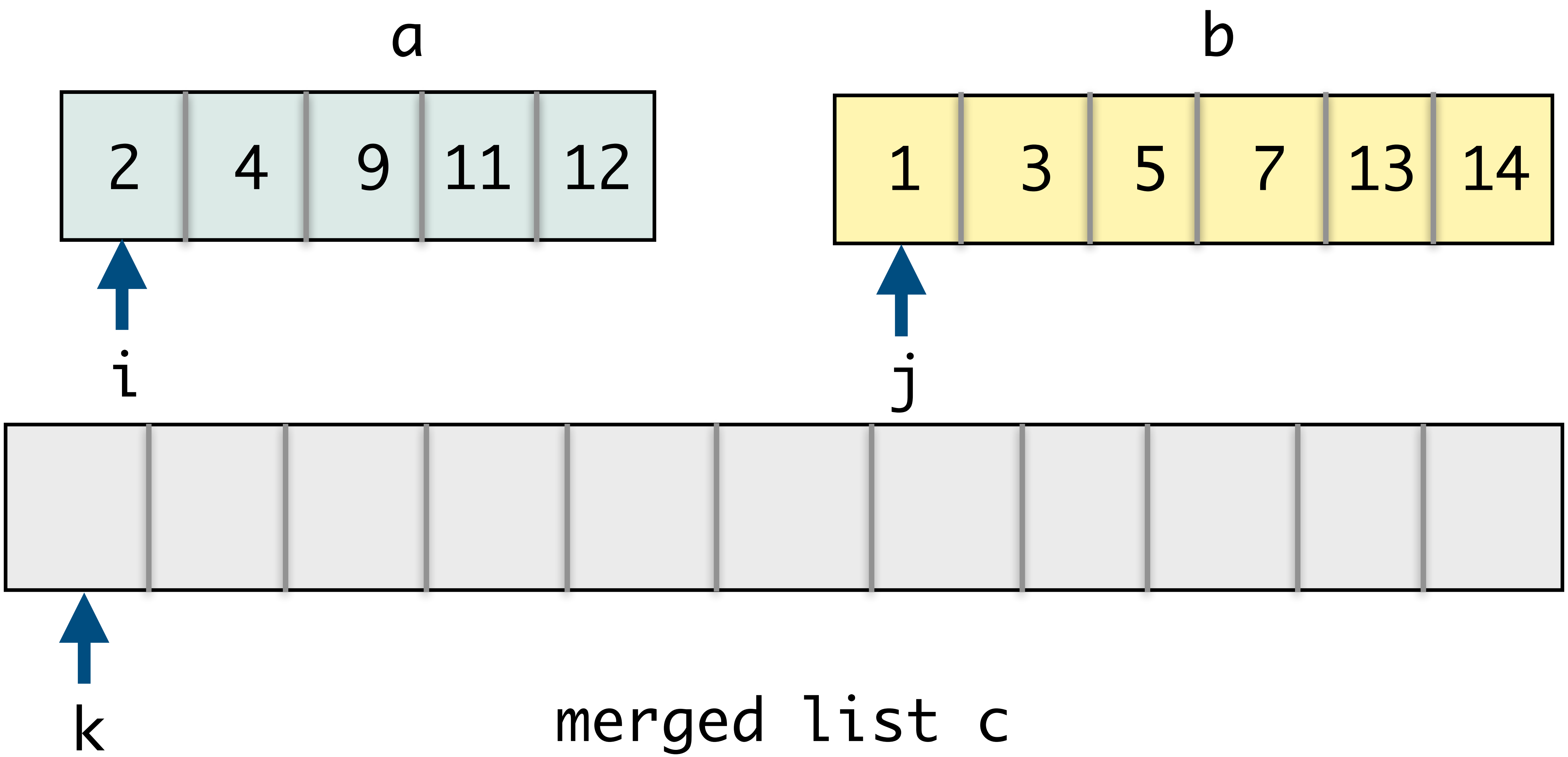
- **Problem.** Given two sorted lists `a` and `b`, how quickly can we merge them into a single sorted list?



Merging Sorted Lists

Is `a[i] <= b[j]` ?

- Yes, `a[i]` appended to `c`
- No, `b[j]` appended to `c`



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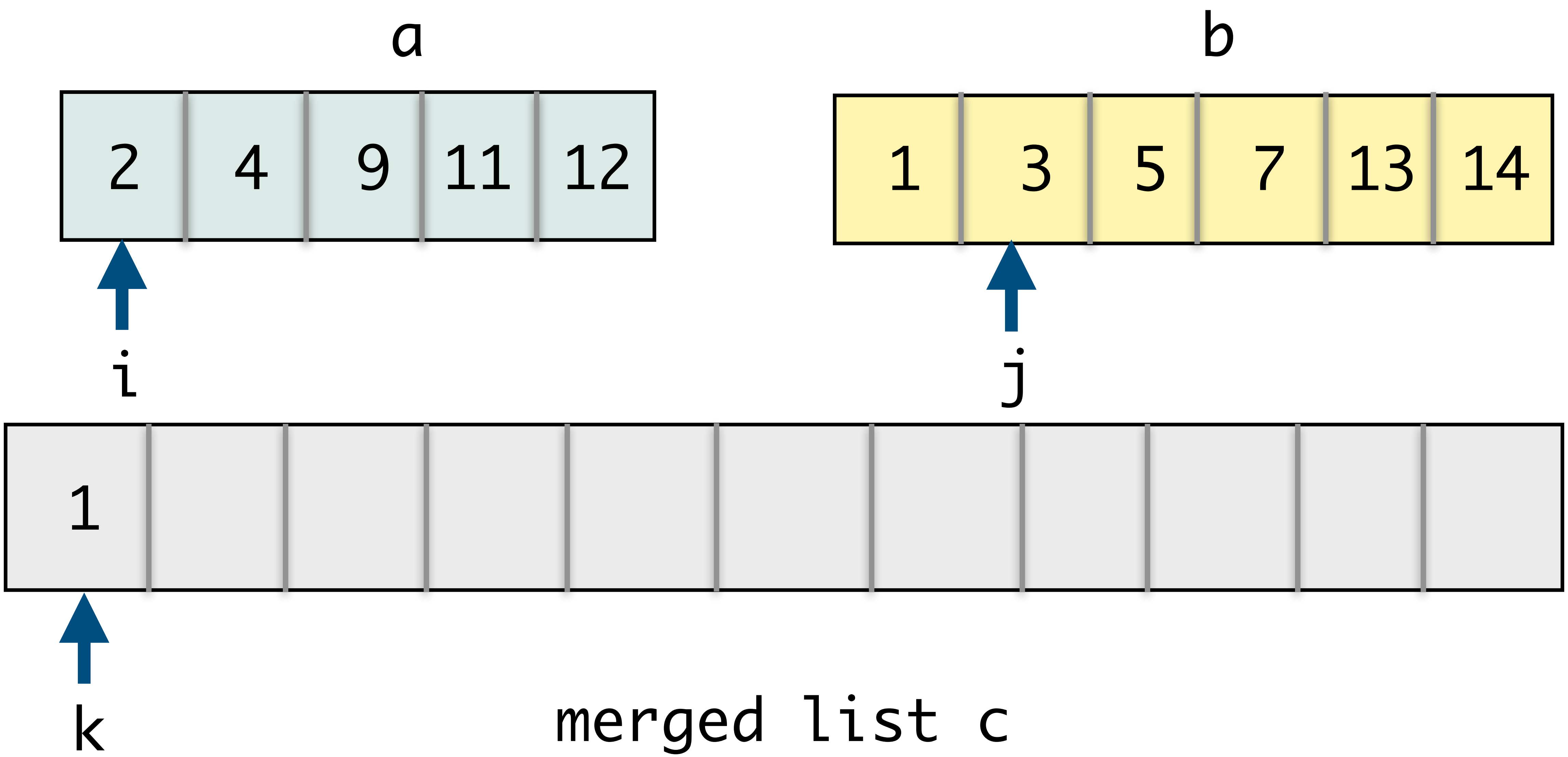

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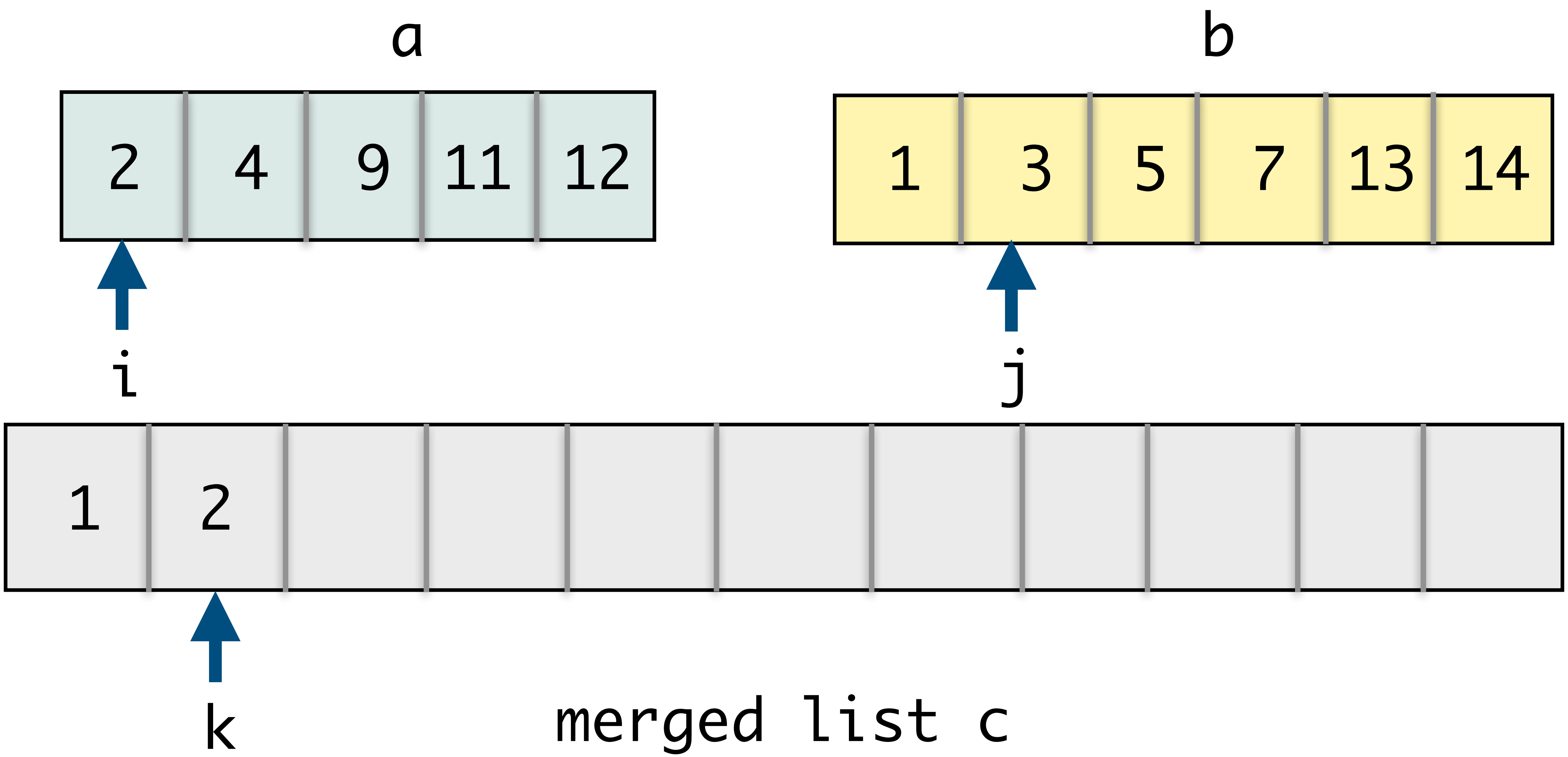

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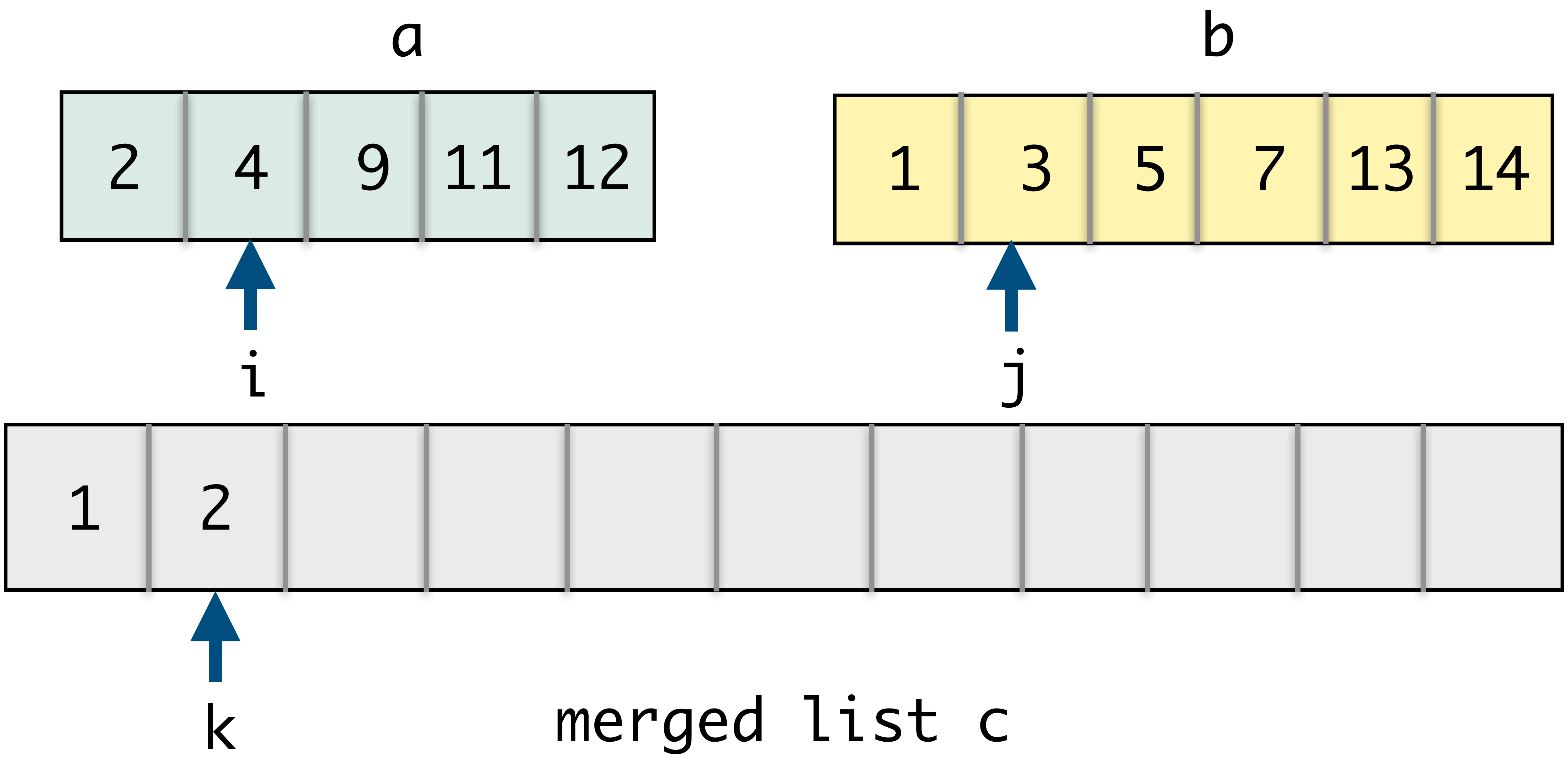

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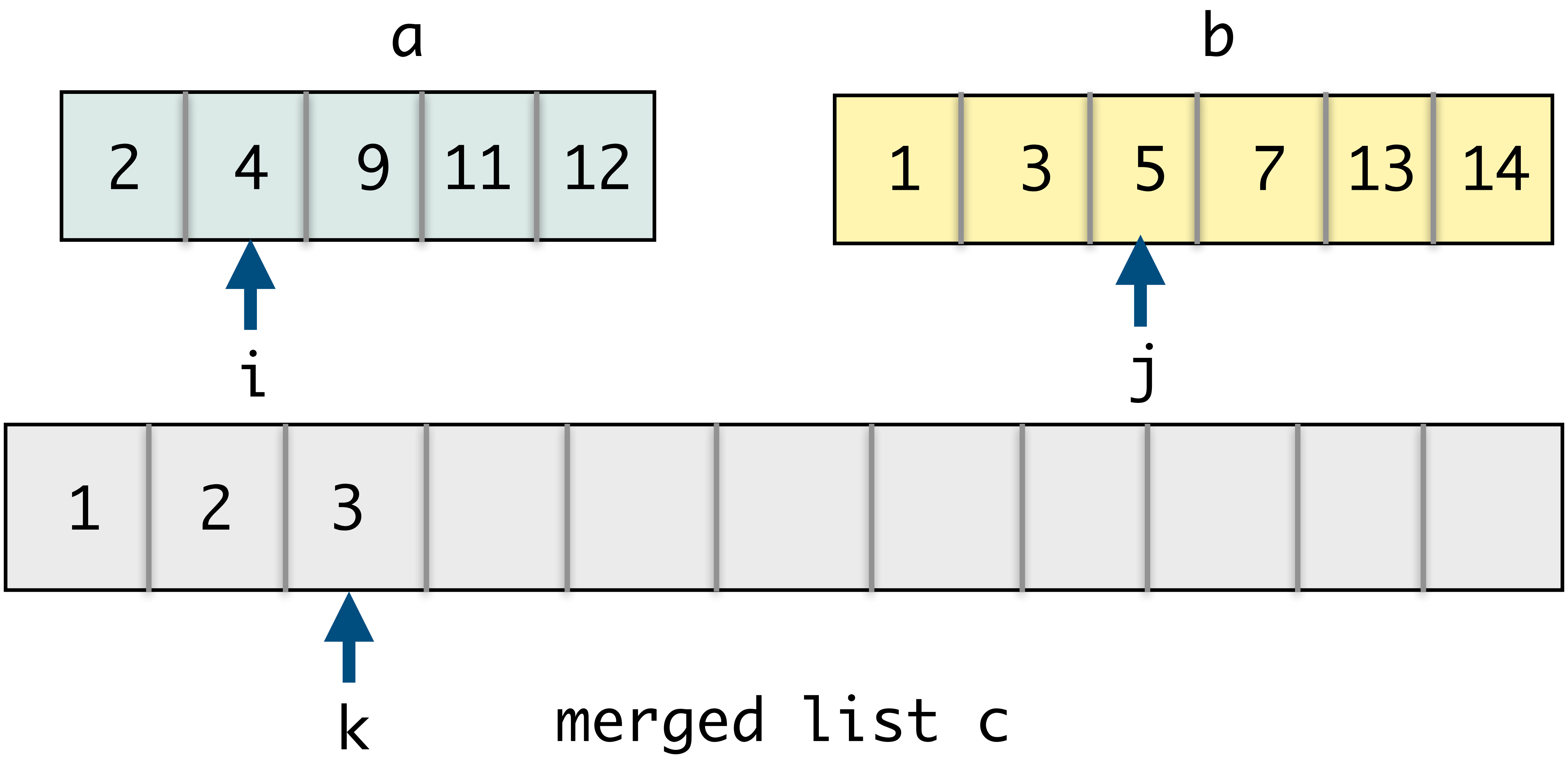

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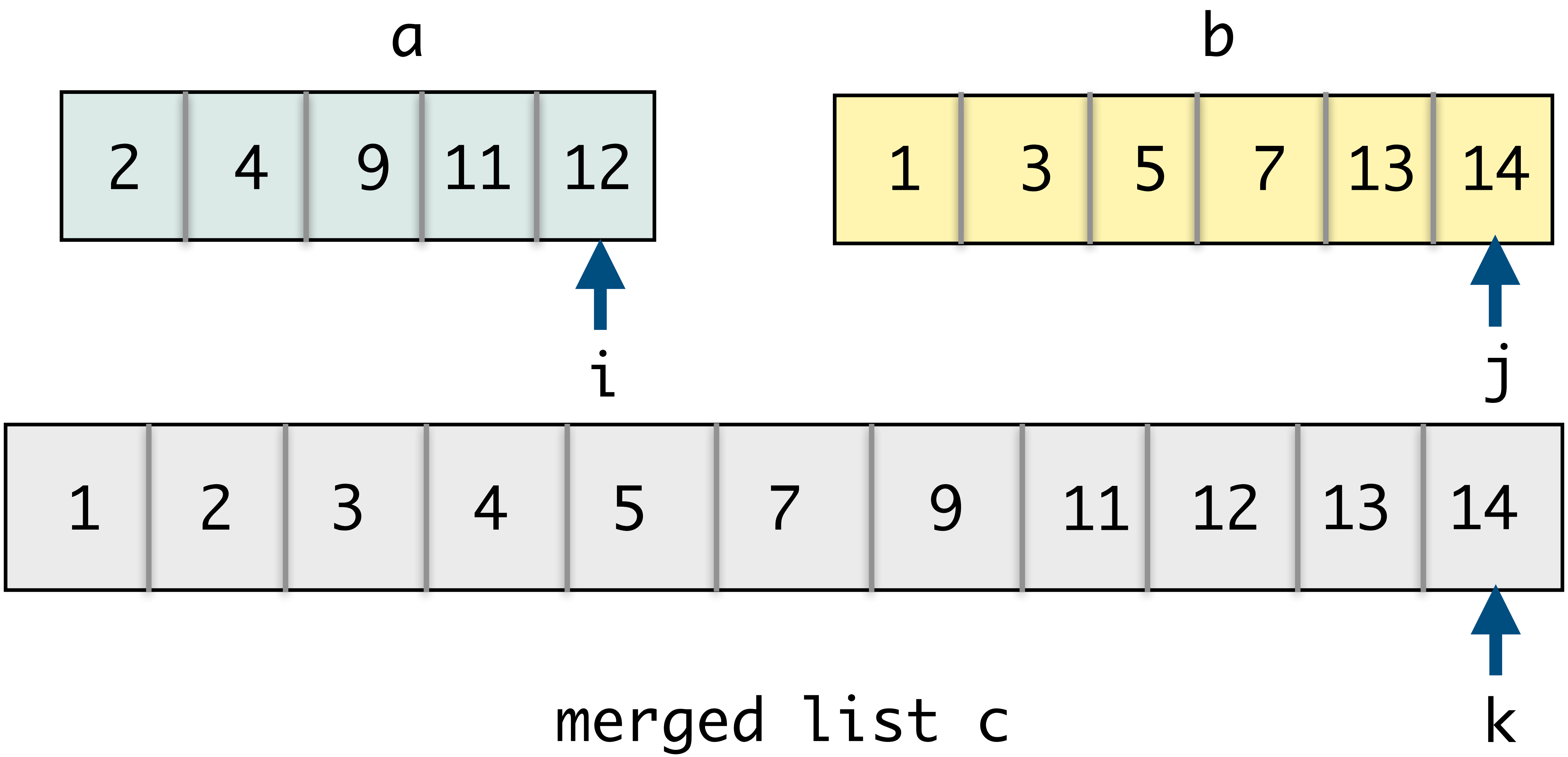


Merging Sorted Lists

- Walk through lists a, b, c maintaining current position of indices i, j, k
- Compare $a[i]$ and $b[j]$, whichever is smaller gets put in the spot of $c[k]$
- Merging two sorted lists into one is an $O(n)$ step algorithm!
- Can use this merge procedure to design our recursive merge sort algorithm!

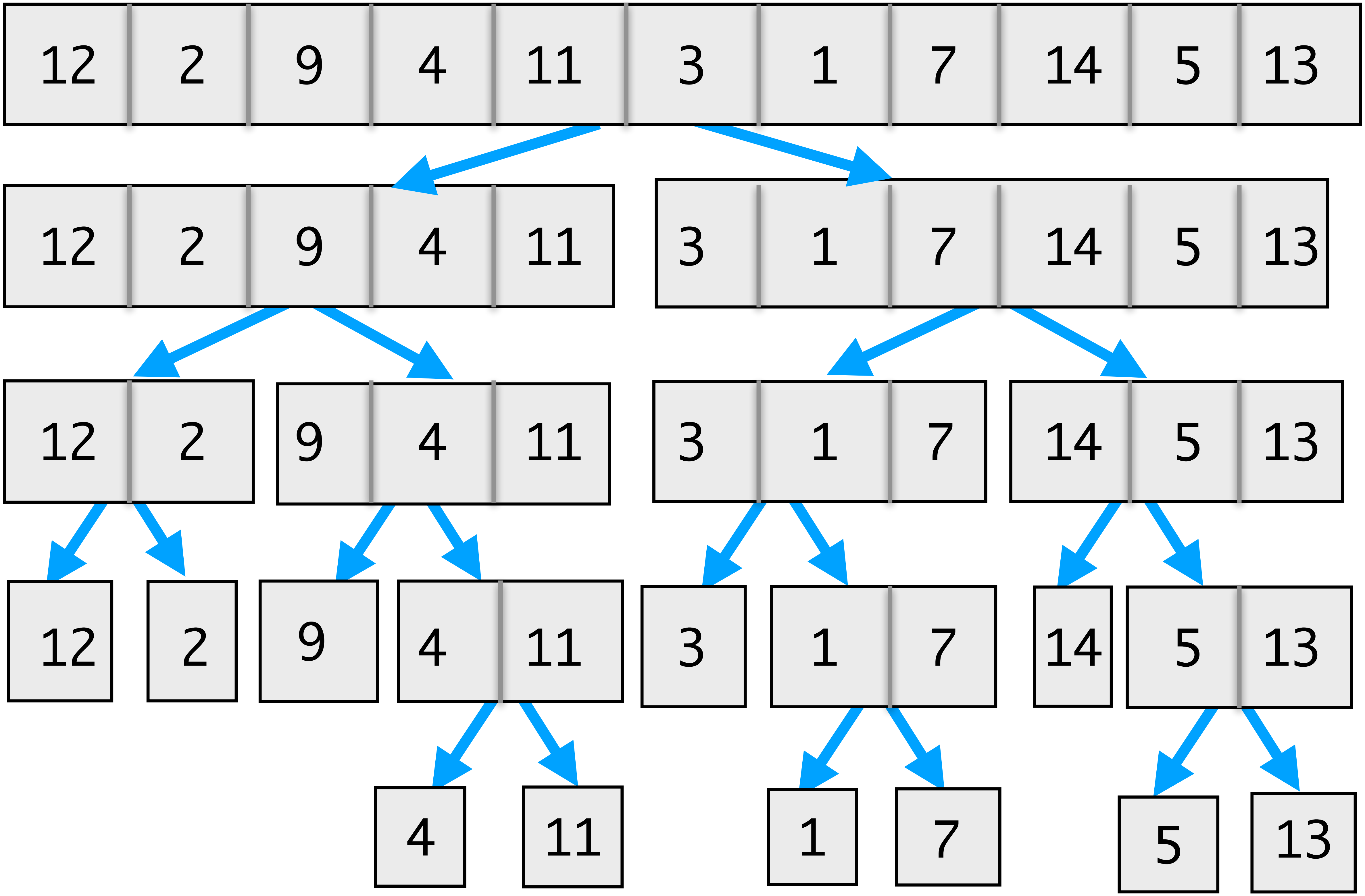
```
def merge(a, b):
    """Merges two sorted lists a and b,
    and returns new merged list c"""
    # initialize variables
    i, j, k = 0, 0, 0
    len_a, len_b = len(a), len(b)
    c = []
    # traverse and populate new list
    while i < len_a and j < len_b:

        if a[i] <= b[j]:
            c.append(a[i])
            i += 1
        else:
            c.append(b[j])
            j += 1

    # handle remaining values
    if i < len_a:
        c.extend(a[i:])

    elif j < len_b:
        c.extend(b[j:])

    return c
```

Merge Sort Algorithm

- **Base case:** If list is empty or contains a single element: it is already sorted
- **Recursive case:**
  - Recursively sort left and right halves
  - Merge the sorted lists into a single list and return it
- **Question:**
  - Where is the ***sorting*** actually taking place?

```python
def merge_sort(lst):
    """Given a list lst, returns
    a new list that is lst sorted
    in ascending order."""
    n = len(lst)

    # base case
    if n == 0 or n == 1:
        return lst

    else:
        m = n//2 # middle

        # recurse on left & right half
        sort_lt = merge_sort(lst[:m])
        sort_rt = merge_sort(lst[m:])

        # return merged list
        return merge(sort_lt, sort_rt)
```

Merge Sort Example



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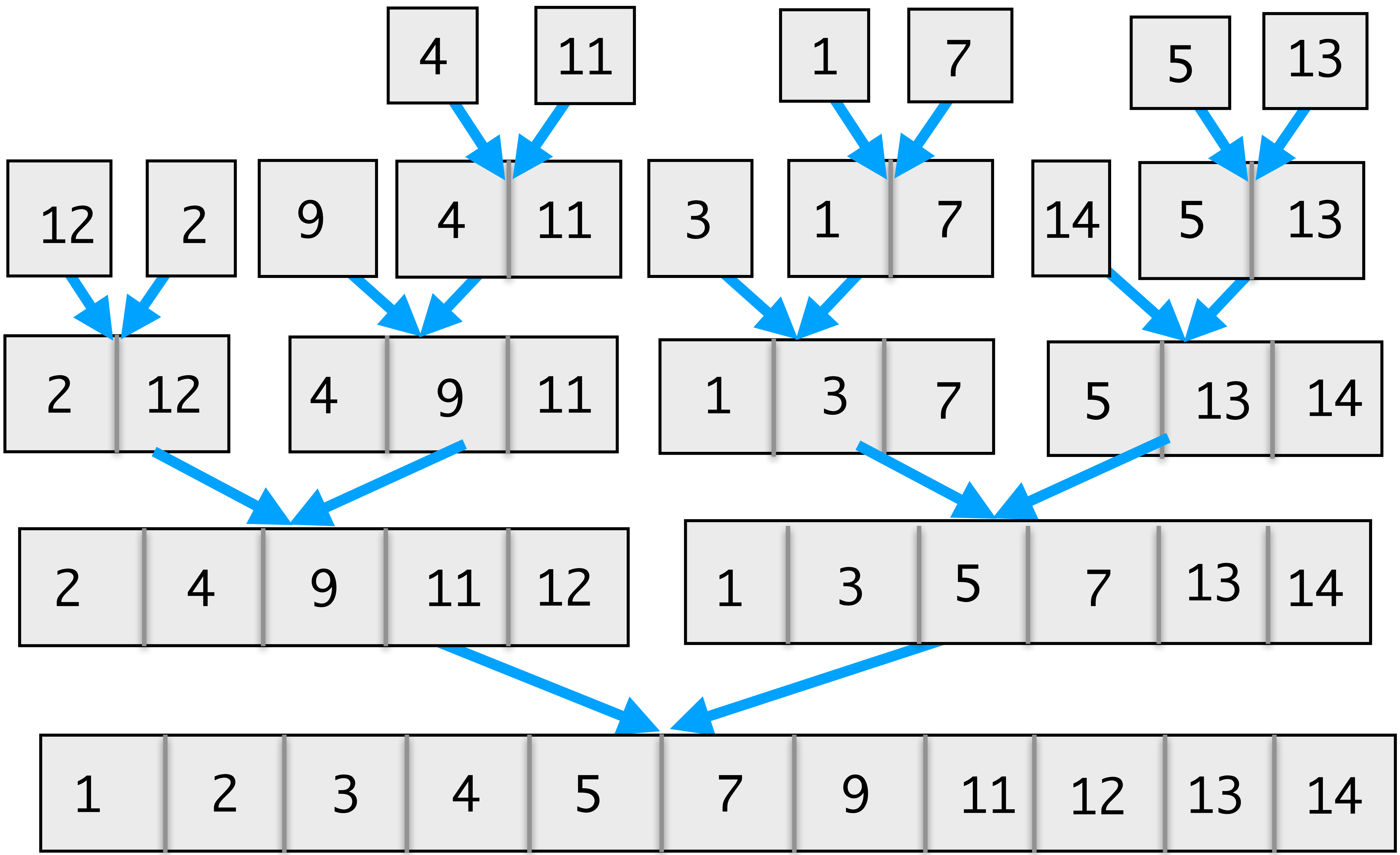

MERGE SÖRT

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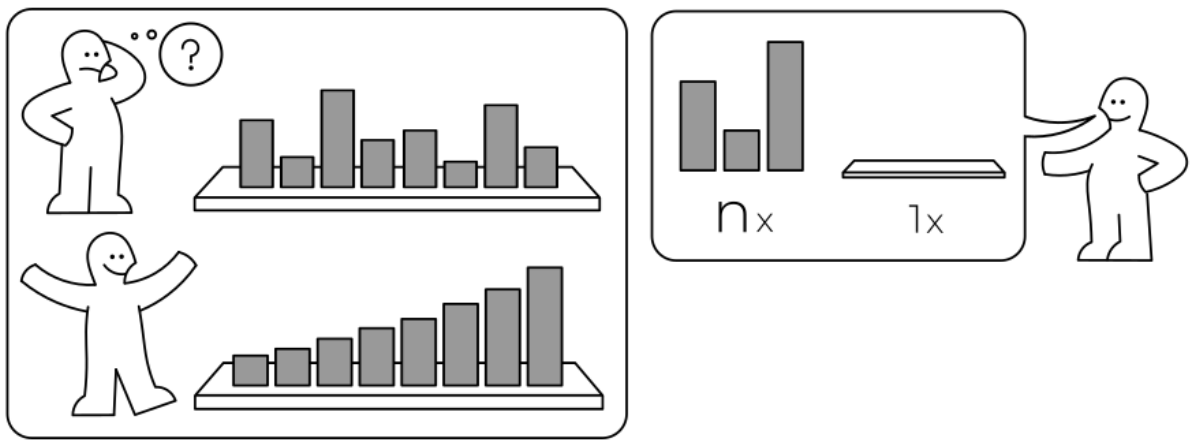

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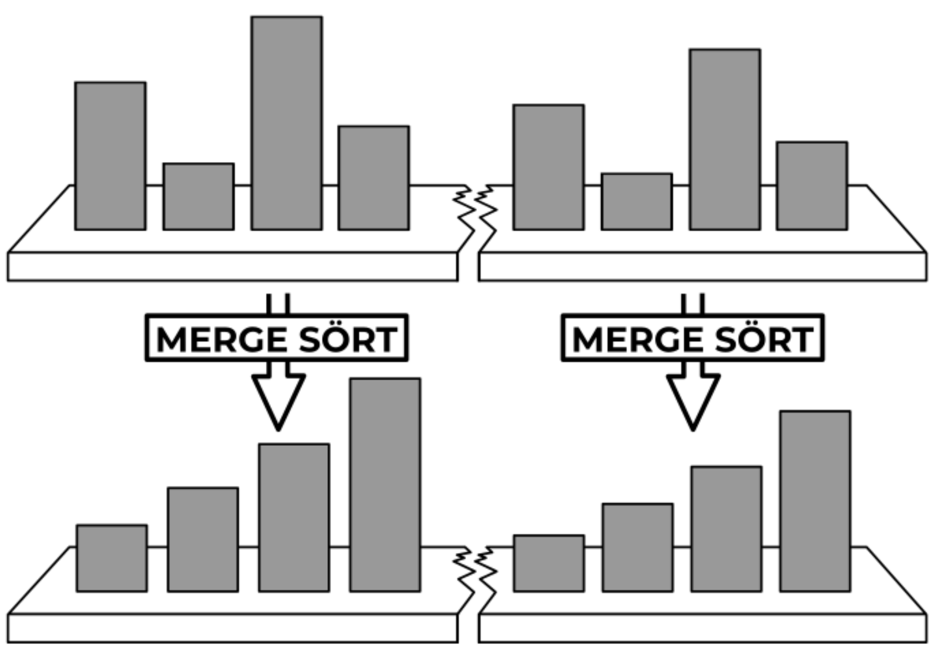

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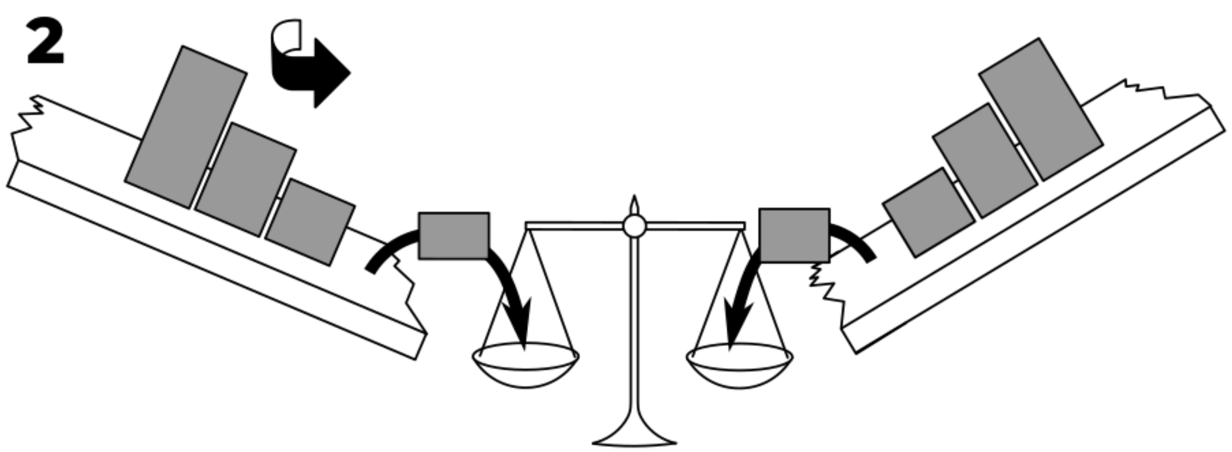

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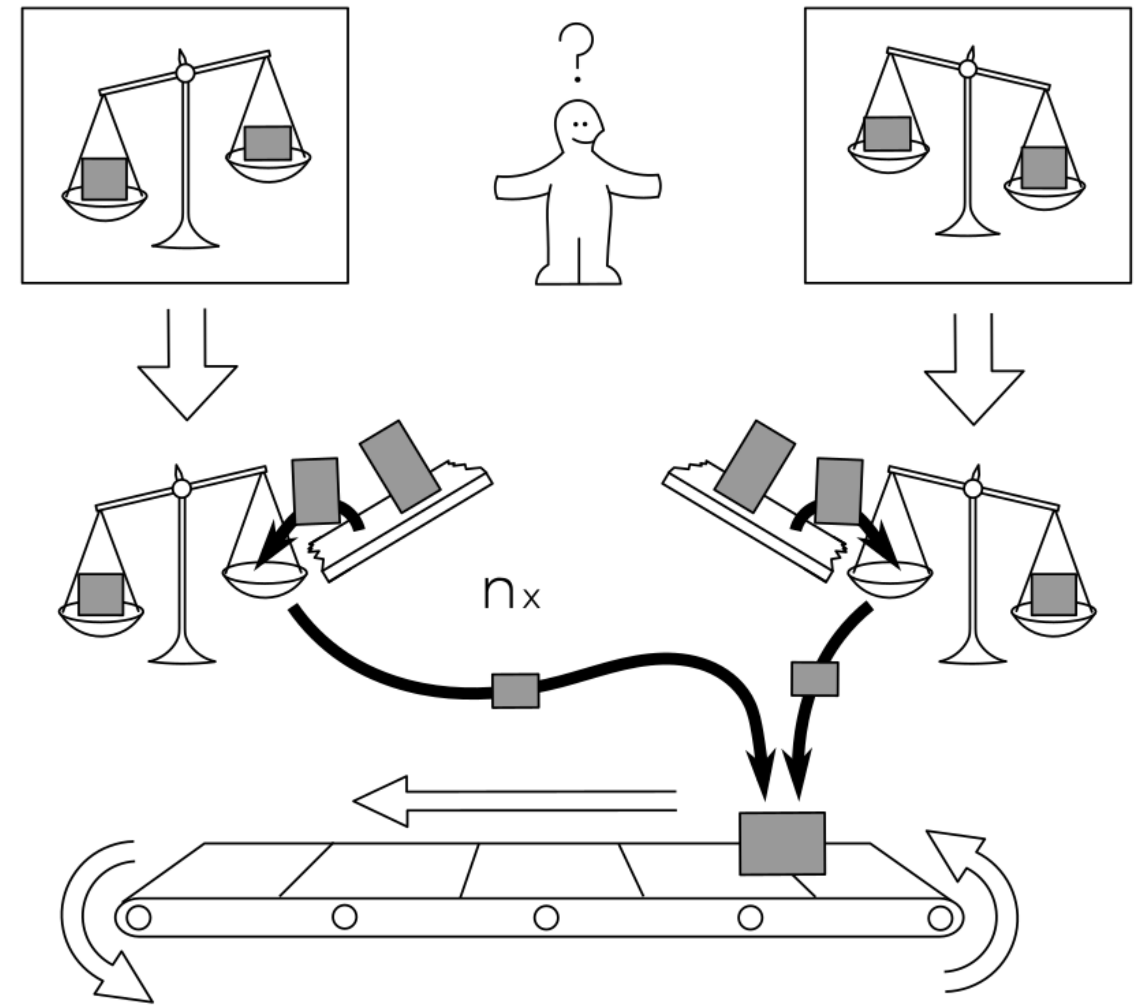
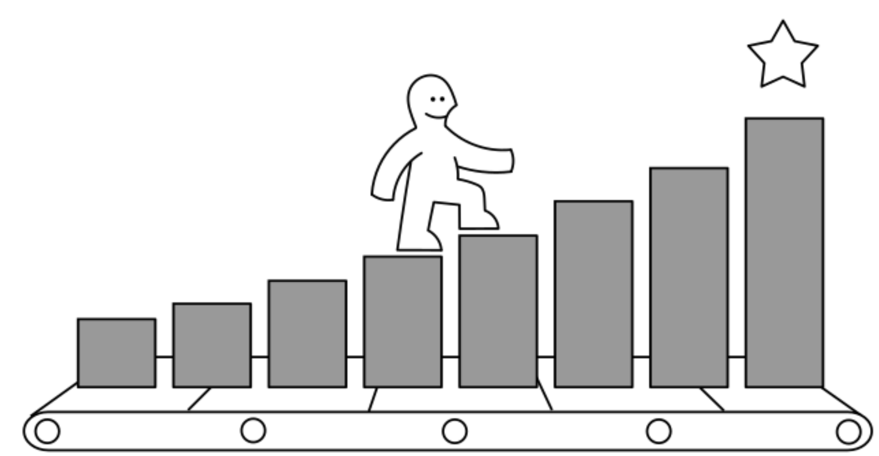
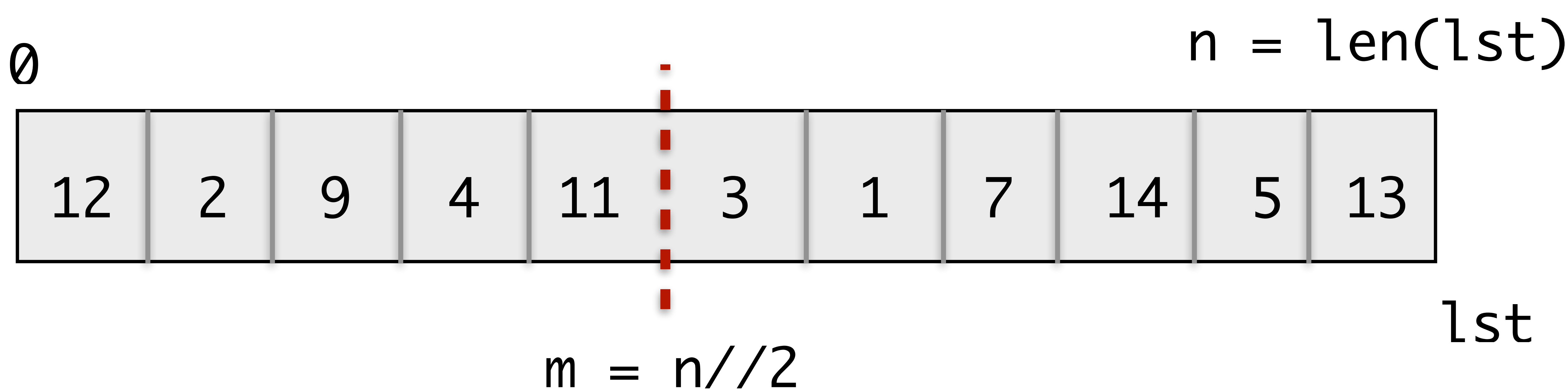


4



Merge Sort Analysis: Basic Idea

- If we split the list in half, sorting the left and right half are smaller versions of the same problem
- **Algorithm Analysis Rough Idea:**
  - **(Divide)** Recursively sort left and right half: happens $\log n$ times
  - **(Unite)** Merge the sorted halves into a single sorted list: takes $O(n)$ times to merge two lists of n items



Big Oh Comparisons

- Selection sort: $O(n^2)$
- Merge sort: $O(n \log n)$