CS134 Lecture 32:
Searching (& Sorting)

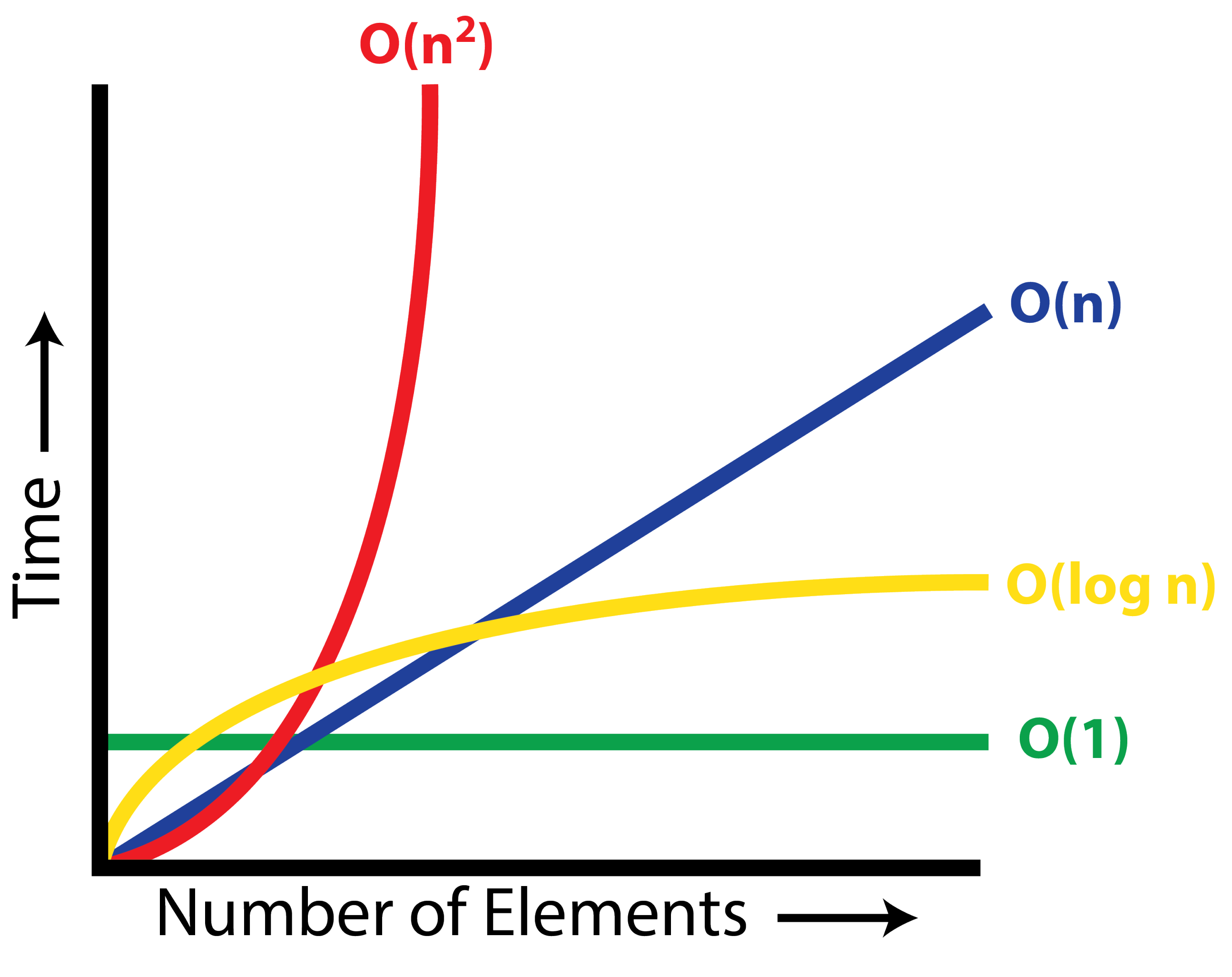
Announcements & Logistics

- **HW 10** due Mon @ 10 pm
  - Last HW on efficiency and Big Oh (Q5 updated with small fix)
- **Lab 8** graded feedback will be returned soon
- **Lab 10** will be released today
  - Very short lab on searching and sorting (today's lecture)
  - No prelab
  - Individual lab but can discuss strategies with lab mate
- CS134 Scheduled Final: **Friday, May 17, 9:30 AM**
  - Room: **TCL 123 (Wege Auditorium) ***

Do You Have Any Questions?

Last Time: Efficiency

- Measured efficiency as number of steps taken by algorithm on worst-case inputs of a given size
- Introduced Big-O notation: captures the rate at which the number of steps taken by the algorithm grows wrt size of input n, "as n gets large"



Today: Searching (and Sorting)

- Discuss recursive implementation of binary search
- Discuss some classic sorting algorithms:
    - ***Selection sorting*** in $O(n^2)$ time
    - A brief (high level) discussion of how we can improve it to $O(n \log n)$
    - Overview of recursive ***merge sort*** algorithm

Searching in a Sequence

Search

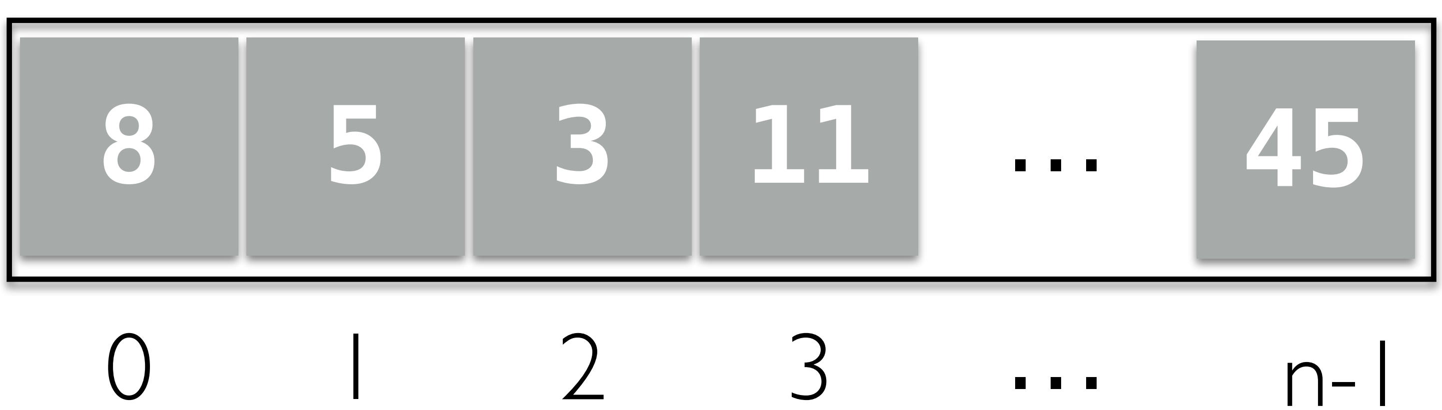
- **Search.** Given an input sequence `seq`, search if a given `item` is in the sequence.
  - For example, if a name is in a sequence of student names
- **Input:** a sequence of n items and a query item
  - For now suppose this can be in ***any order***
- **Output:** True if query item is in sequence, else False
- Can use `in` operator to do this (calls `__contains__`)
  - But without knowing how it works, can't analyze efficiency
- Let's figure out a direct way to solve this problem

Searching in a Sequence

- First algorithm: iterate through the items in sequence and compare each item to query

```python
def linear_search(item, seq):
    for elem in seq:
        if elem == item:
            return True
    return False
```

Might return early if item is first elem in seq, but we are interested in the **worst case analysis**; worst case is if item is not in seq at all



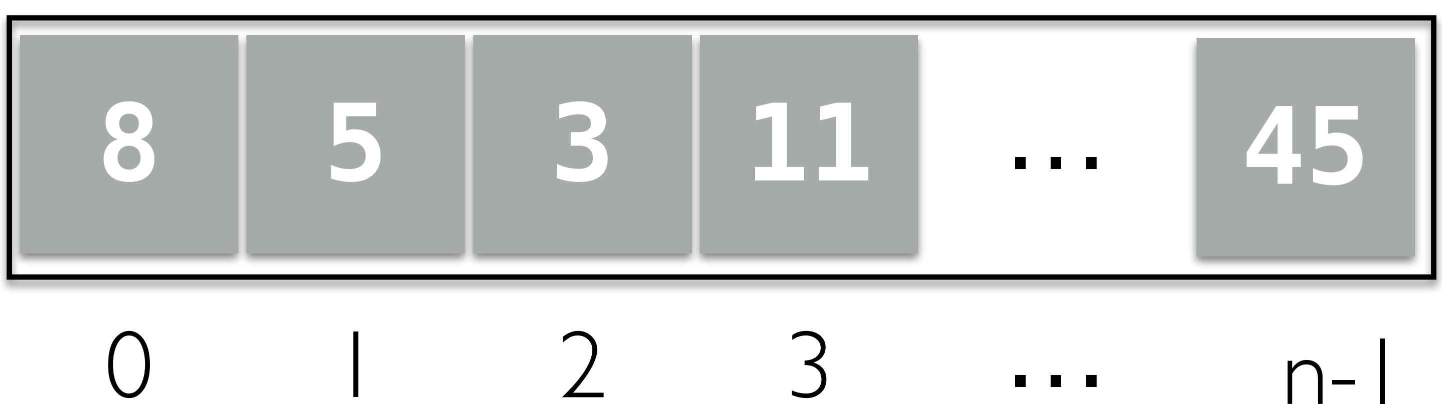
Searching in a Sequence

- In the worst case, we have to walk through the entire sequence
- Overall, the number of steps is linear in n : we write $O(n)$ in Big Oh

```python
def linear_search(item, seq):
    for elem in seq:
        if elem == item:
            return True
    return False
```

Loop runs n items in worst case

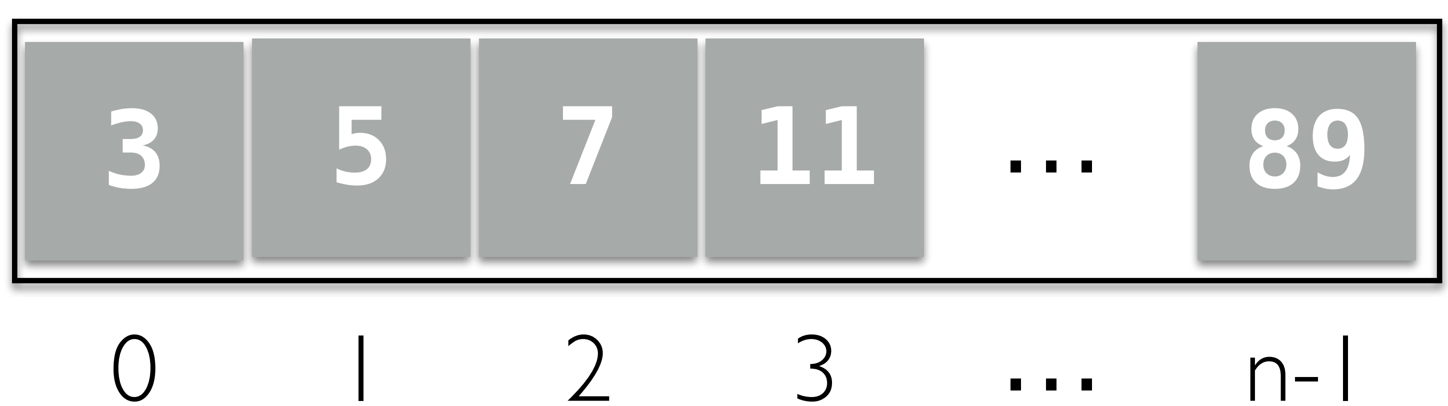
One equality check per iteration: assume comparing elem == item is one step



Searching in an Array

- Can we do better?
    - Not if the elements are in arbitrary order
- What if the sequence is **sorted**?
    - Can we utilize this somehow and search more efficiently?

How do we search for an item (say 10) in a **sorted** array?



Let’s Play a Game

- I’m thinking of a number between 0 and 100…
  - If you guess a number, I’ll tell you either:
    - You’ve guessed my number!
    - My number is larger than your guess
    - My number is smaller than your guess
- What is your guessing strategy?

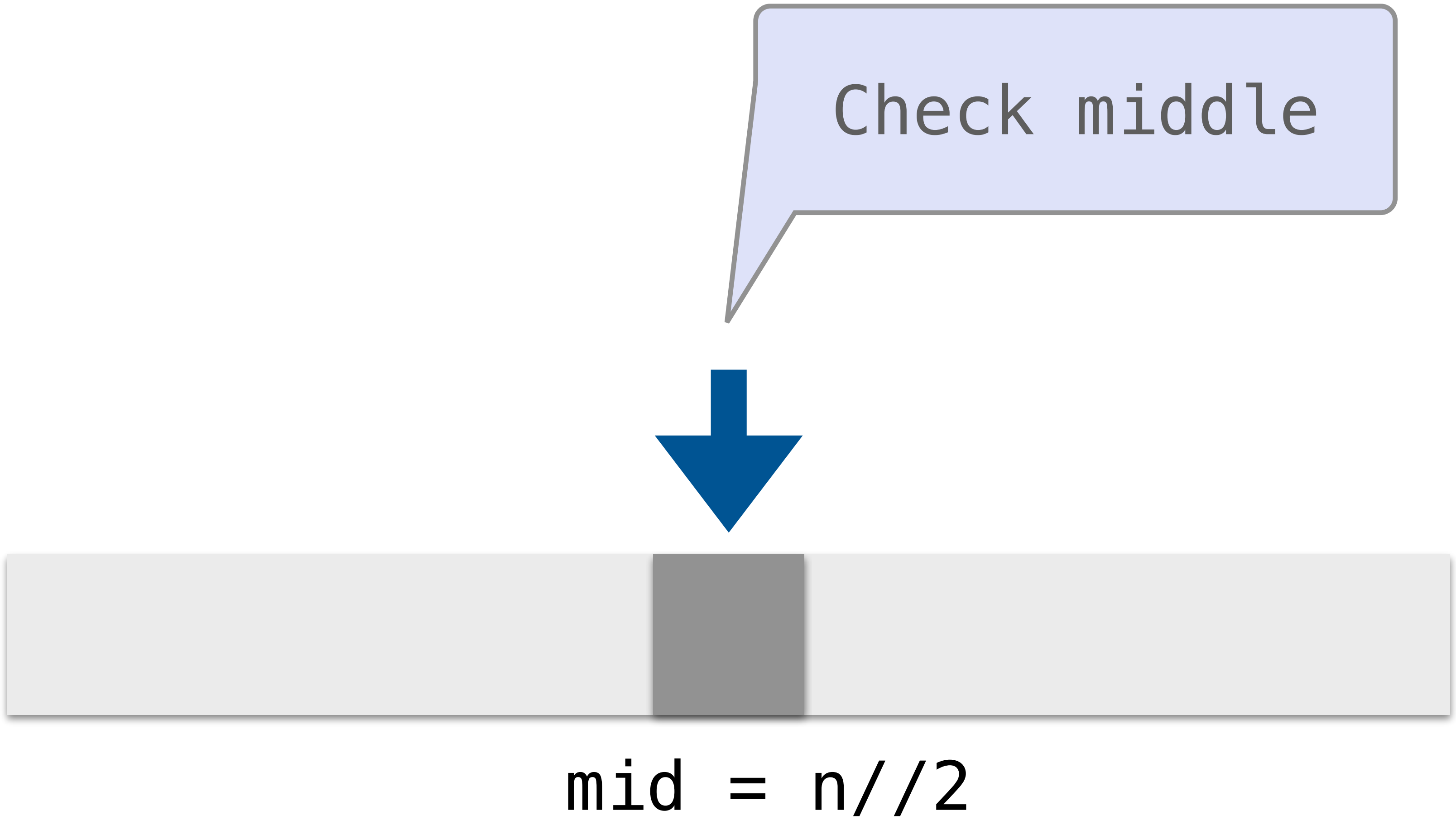
- What if I picked a number between 0 and 1 million?

Binary Search

- The **search algorithm** we just discussed to guess a number can be used search in a sorted list. It's called **binary search**
- It can be much more efficient than a **linear search**
  - Takes $\approx \log n$ lookups if we can index into sequence efficiently
- Which data structure supports fast access/indexing?
  - Accessing an item at index i in an array requires constant time
  - Accessing an item at index i in a LinkedList can require traversing the whole list (even if it is sorted!): linear time
- To get a more efficient search algorithm, it is not only important to use the right algorithm, we need to use the right data structure as well!

Binary Search

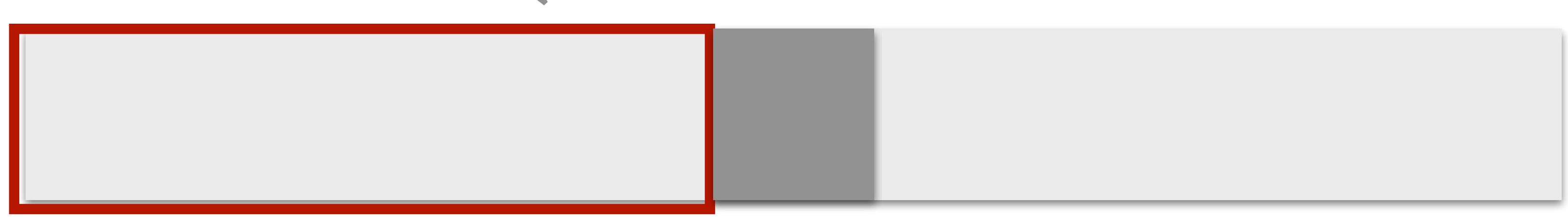
- Base cases? When are we done?
    - If list is too small (or empty) to continue searching, return False
    - If item we're searching for is the middle element, return True



Binary Search

- Recursive case:
    - Recurse on left side if item is smaller than middle
    - Recurse on right side if item is larger than middle

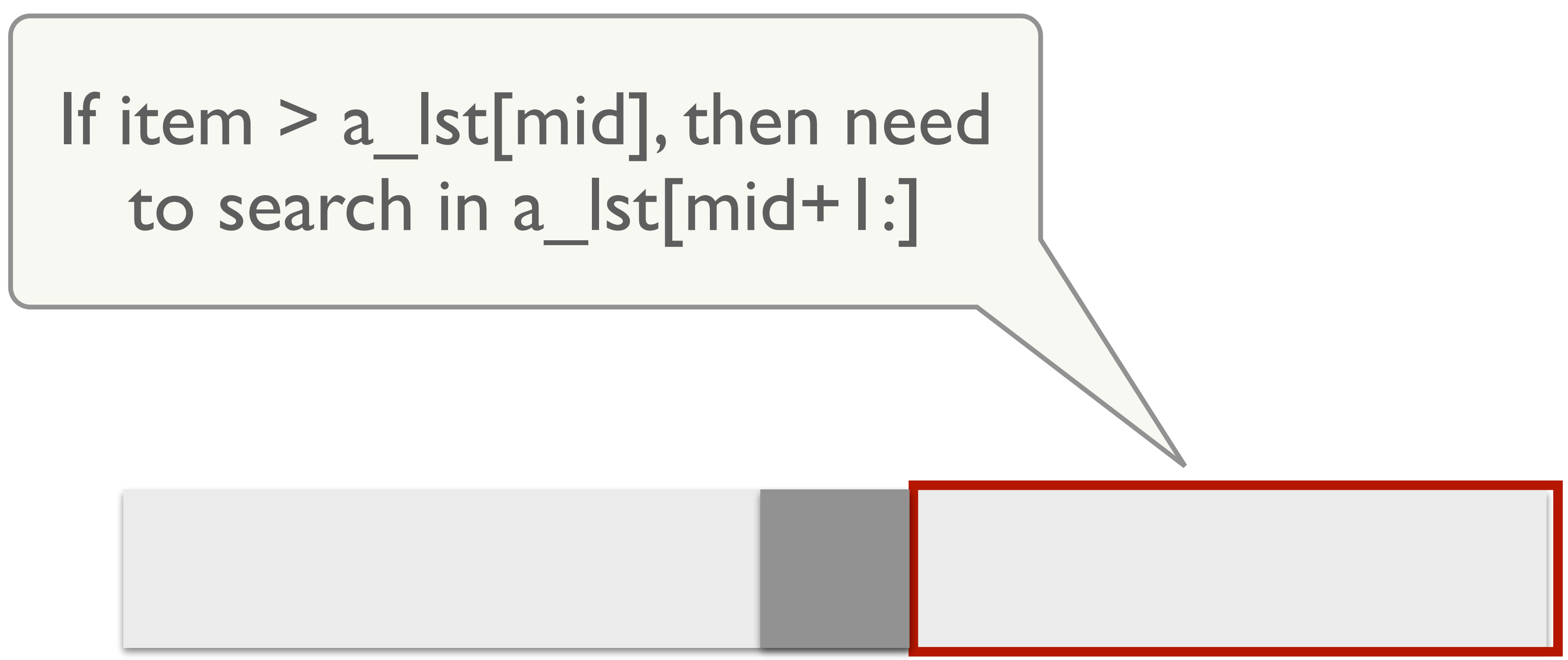
If item < a_lst[mid], then need to search in a_lst[:mid]



`mid = n//2`

Binary Search

- Recursive case:
  - Recurse on left side if item is smaller than middle
  - Recurse on right side if item is larger than middle




```
mid = n//2
```

```python
def binary_search(seq, item):
    """Assume seq is sorted. If item is
    in seq, return True; else return False."""

    n = len(seq)

    # base case 1
    if n == 0:
        return False

    mid = n // 2
    mid_elem = seq[mid]

    # base case 2
    if item == mid_elem:
        return True

    # recurse on left
    elif item < mid_elem:
        left = seq[:mid]
        return binary_search(left, item)

    # recurse on right
    else:
        right = seq[mid+1:]
        return binary_search(right, item)
```

Technically, there is one *small* problem with our implementation. List splicing is actually O(n)!

Binary Search: Improved

```python
def binary_search_helper(seq, item, start, end):
    '''Recursive helper function used in binary search'''

    # base case 1
    if start > end:
        return False

    mid = (start + end) // 2
    mid_elem = seq[mid]

    if item == mid_elem:
        return True

    # recurse on left
    elif item < mid_elem:
        return binary_search_helper(seq, item, start, mid-1)

    # recurse on right
    else:
        return binary_search_helper(seq, item, mid+1, end)

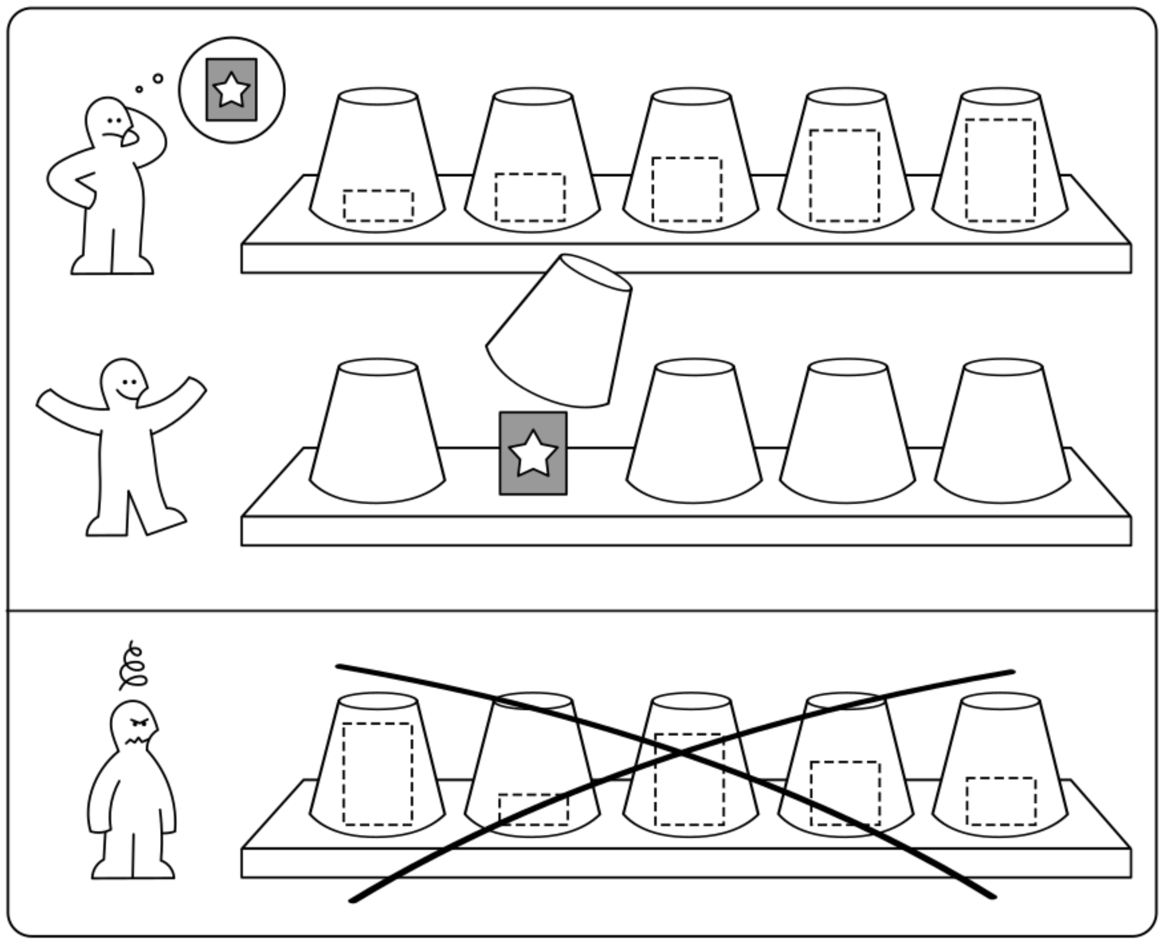

def binary_search_improved(seq, item):

    return binary_search_helper(seq, item, 0, len(seq)-1)
```

Passing start/end indices as arguments avoids the need to splice!

BINÄRY SEARCH

IDEA



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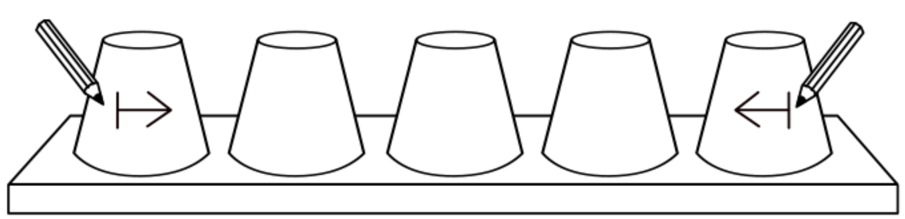

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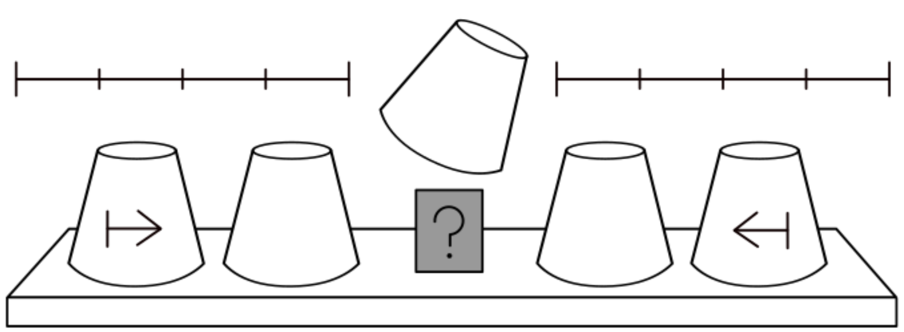

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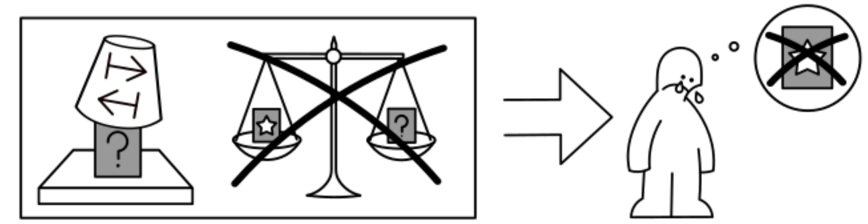

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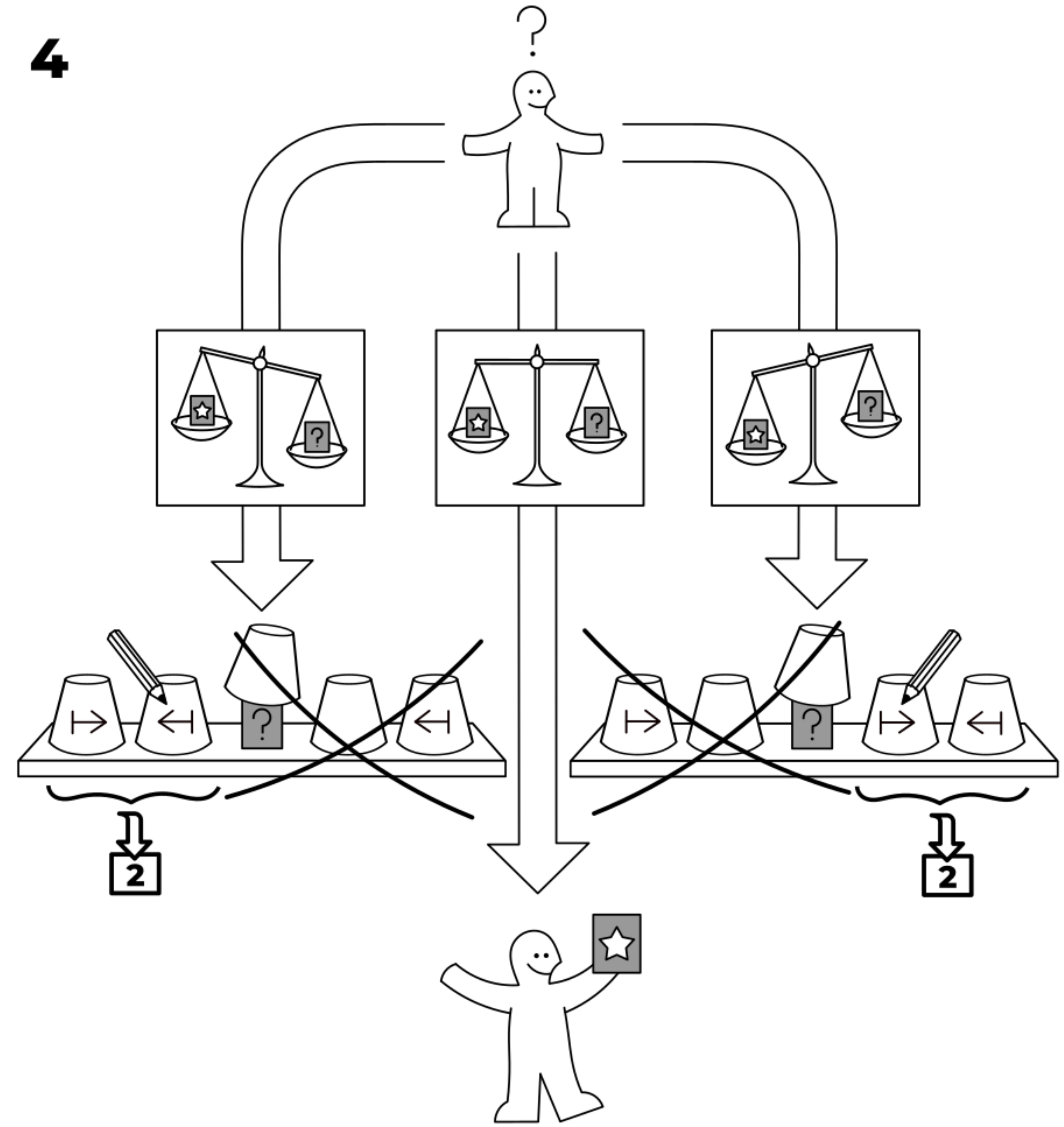


More on Big Oh

Big-O Notation

- Tells you how fast an algorithm is / the run-time of algorithms
  - But not in seconds!
- Tells you how fast the algorithm grows in number of operations

O(log n)

"Big O"   Number of Operations

Understanding Big-O

- Notation: n often denotes the number of elements (size)
- **Constant time** or $O(1)$: when an operation does not depend on the number of elements, e.g.
  - Addition/subtraction/multiplication of two values, or defining a variable etc are all constant time
- **Linear time** or $O(n)$: when an operation requires time proportional to the number of elements, e.g.:

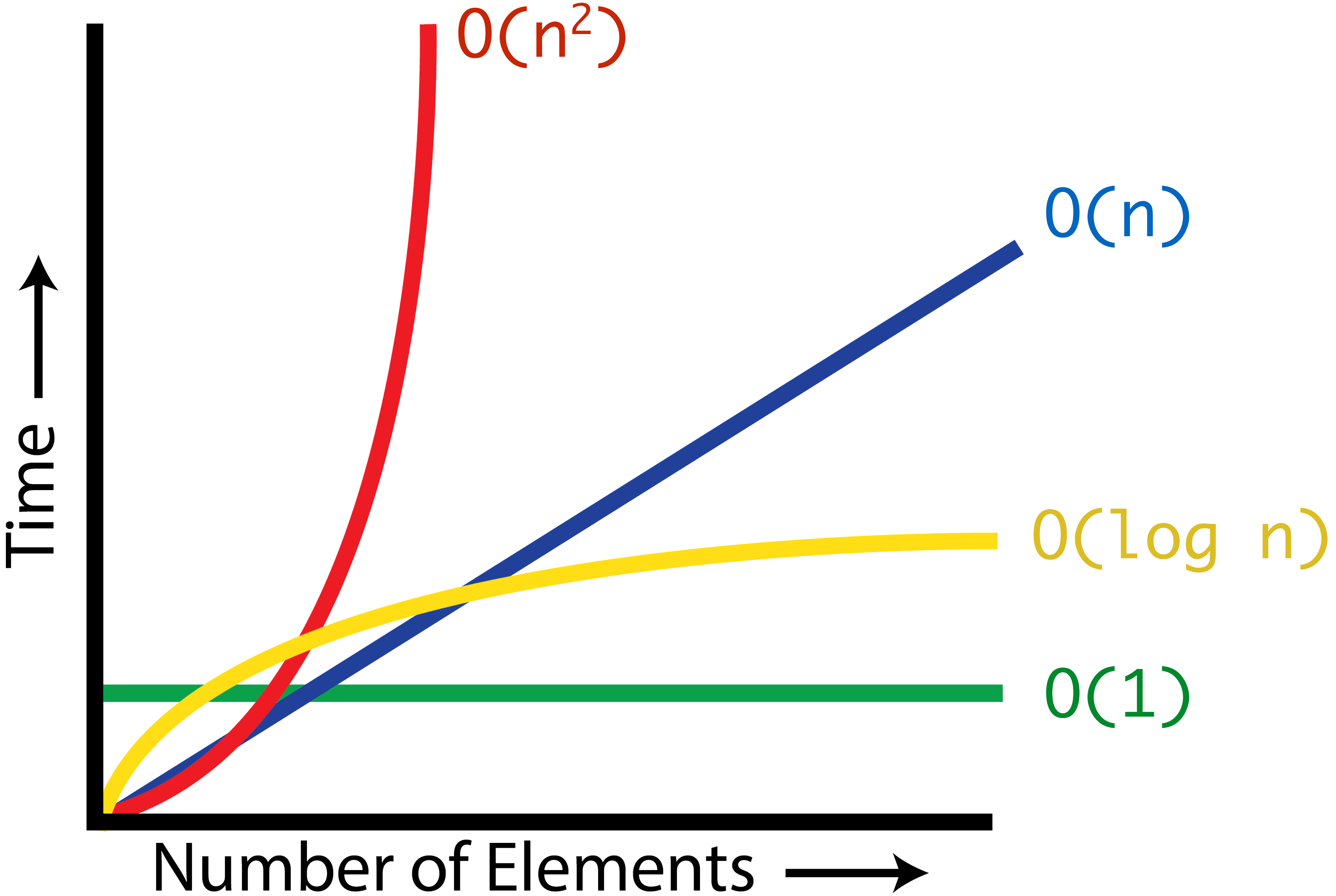
```
for item in seq:
    <do something>
```

- **Quadratic time** or $O(n^2)$: nested loops are often quadratic, e.g.,

```
for i in range(n):
    for j in range(n):
        <do something>
```

Big-O: Common Functions

- Notation: n often denotes the number of elements (size)
- Our goal: understand efficiency of some algorithms at a high level



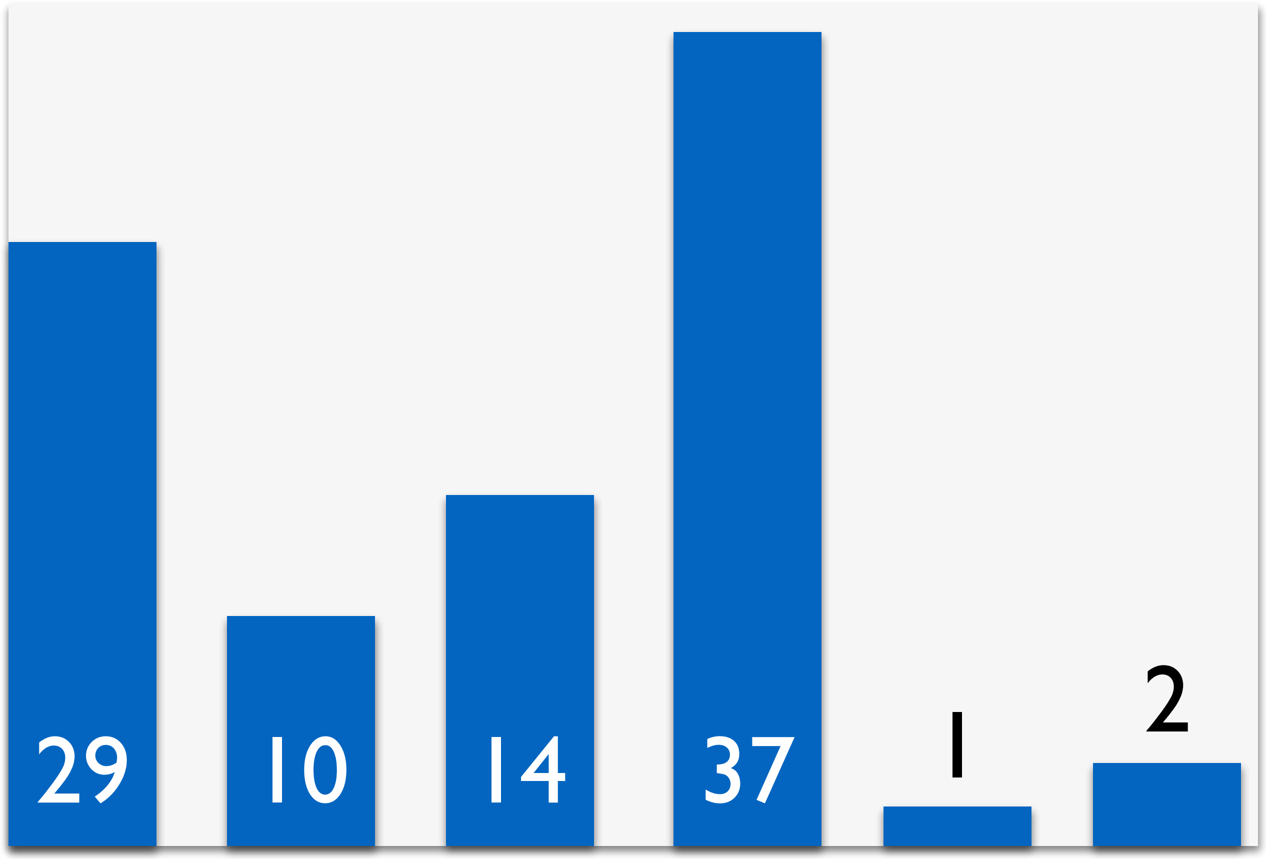
Sorting

Sorting

- **Problem:** Given a sequence of unordered elements, we need to sort the elements in ascending order.
- There are many ways to solve this problem!
- Built-in sorting functions/methods in Python
    - `sorted()`: ***function*** that returns a new sorted list
    - `sort()`: ***list method*** that mutates and sorts the list
- **Today:** how do we design our own sorting algorithm?
- **Question:** What is the best (most efficient) way to sort n items?
- We will use Big-O to find out!

Selection Sort

- A possible approach to sorting elements in a list/array:
  - Find the smallest element and move (swap) it to the first position
  - *Repeat*: find the second-smallest element and move it to the second position, and so on



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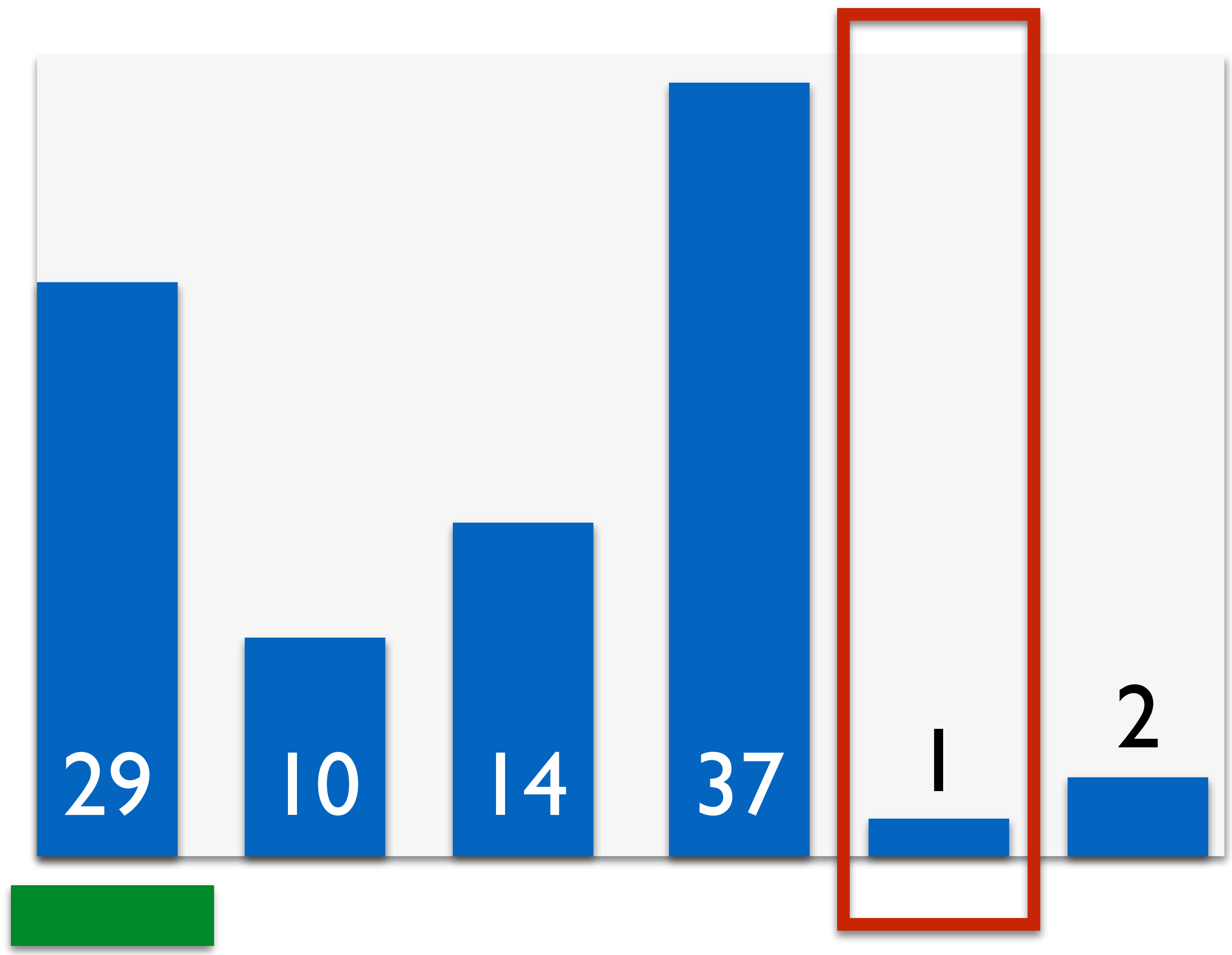

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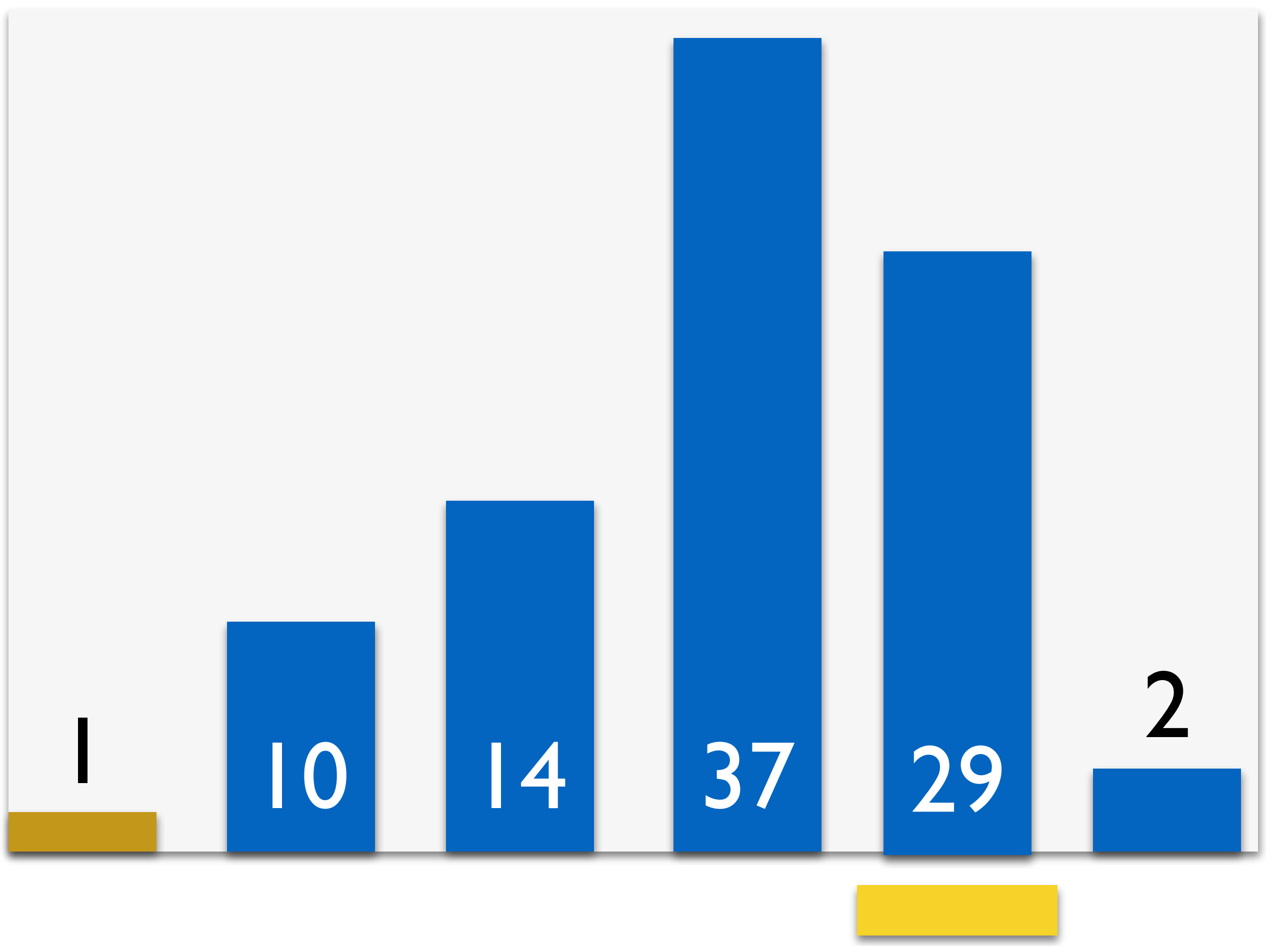

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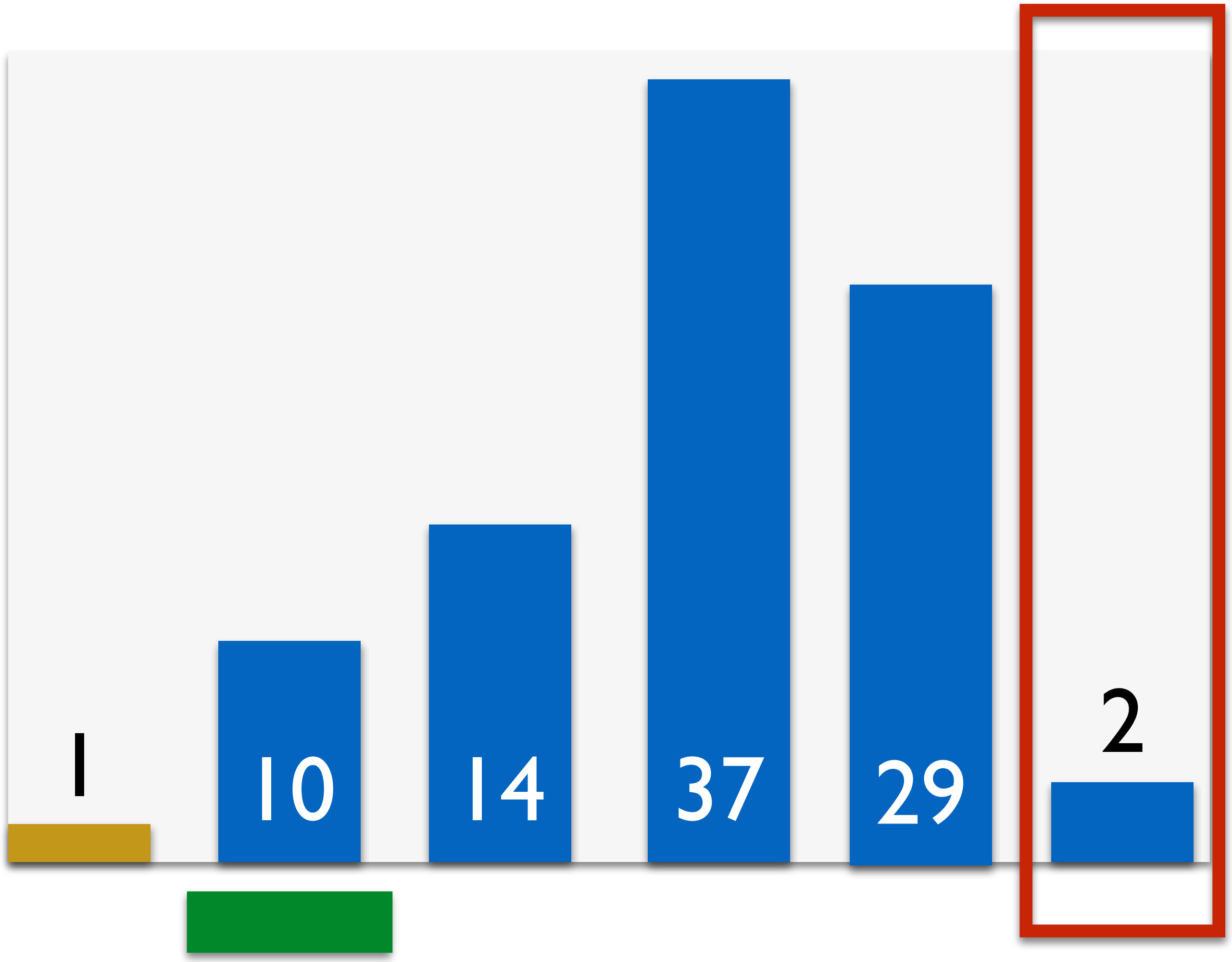
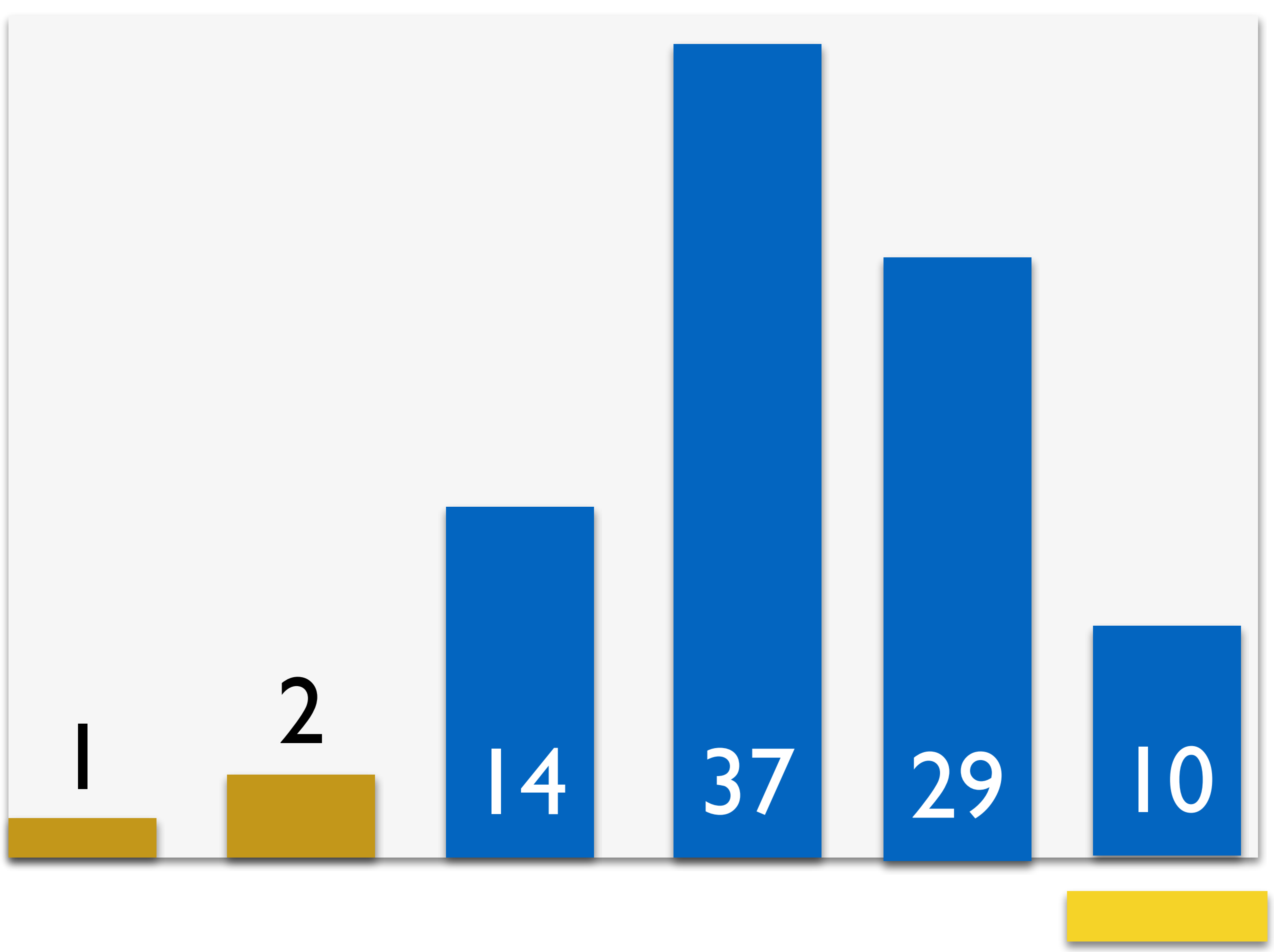


Selection Sort

- Find the smallest element and move (swap) it to the first position
- *Repeat*: find the second-smallest element and move it to the second position, and so on
- The gold bars represent the sorted portion of the list.



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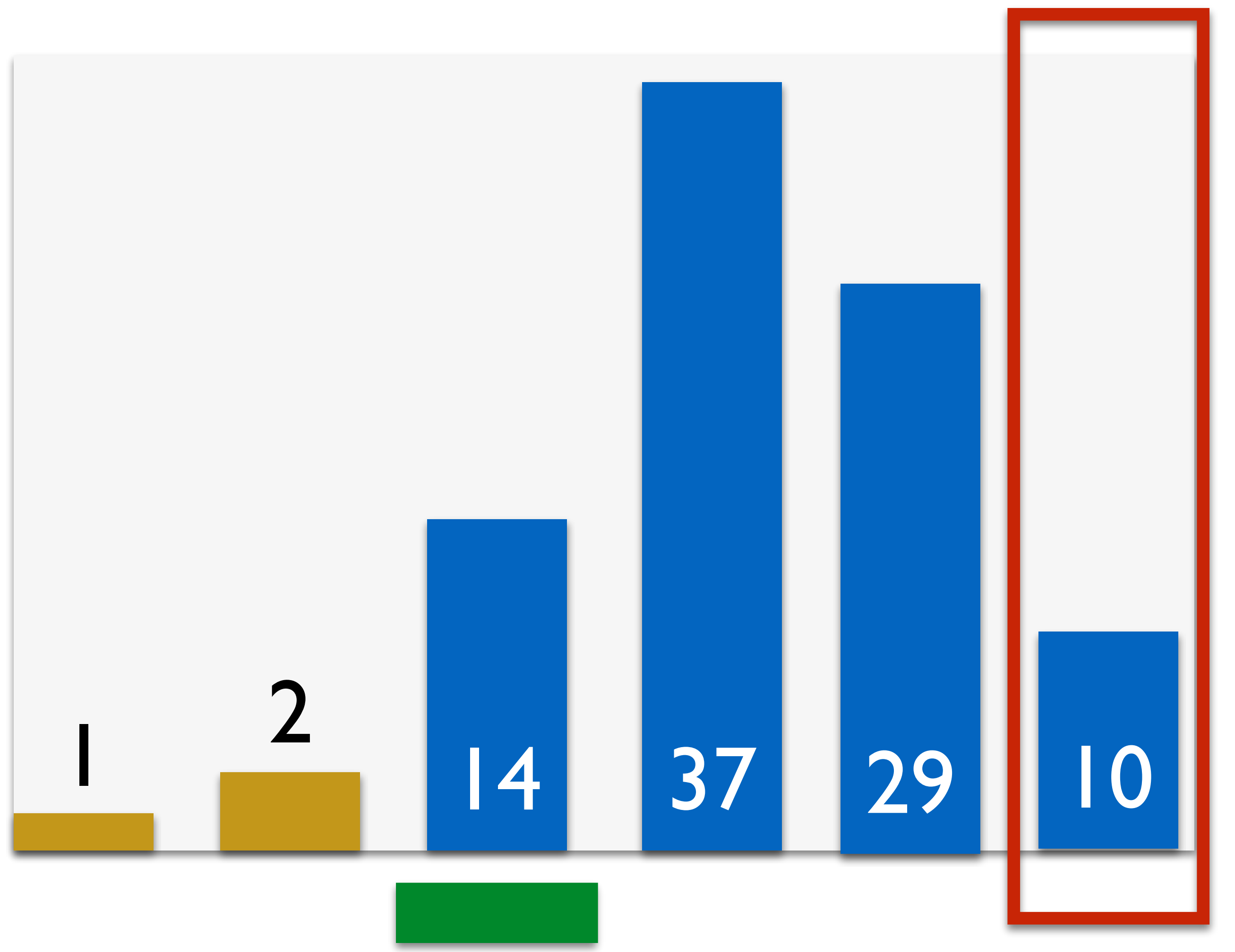

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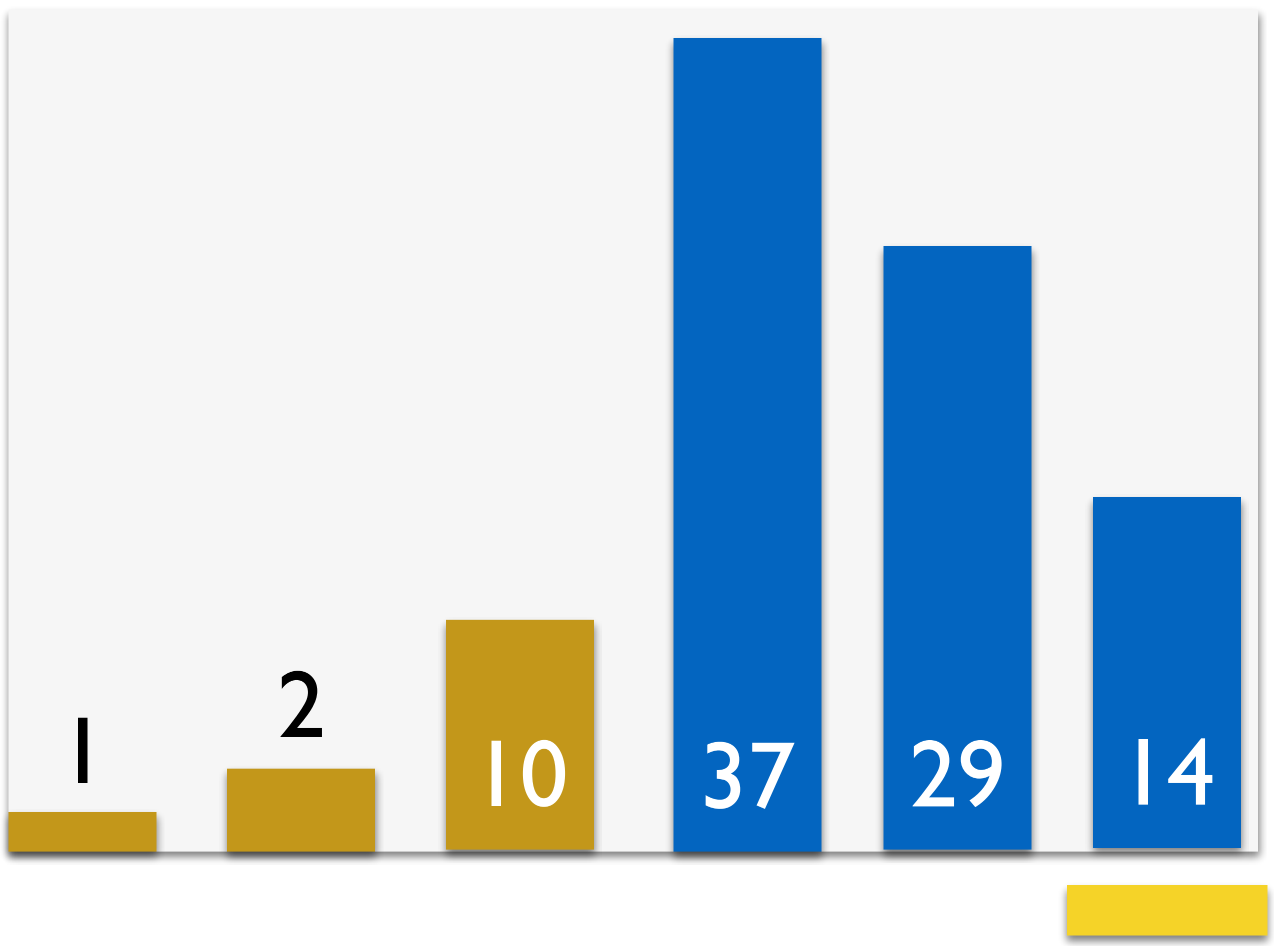

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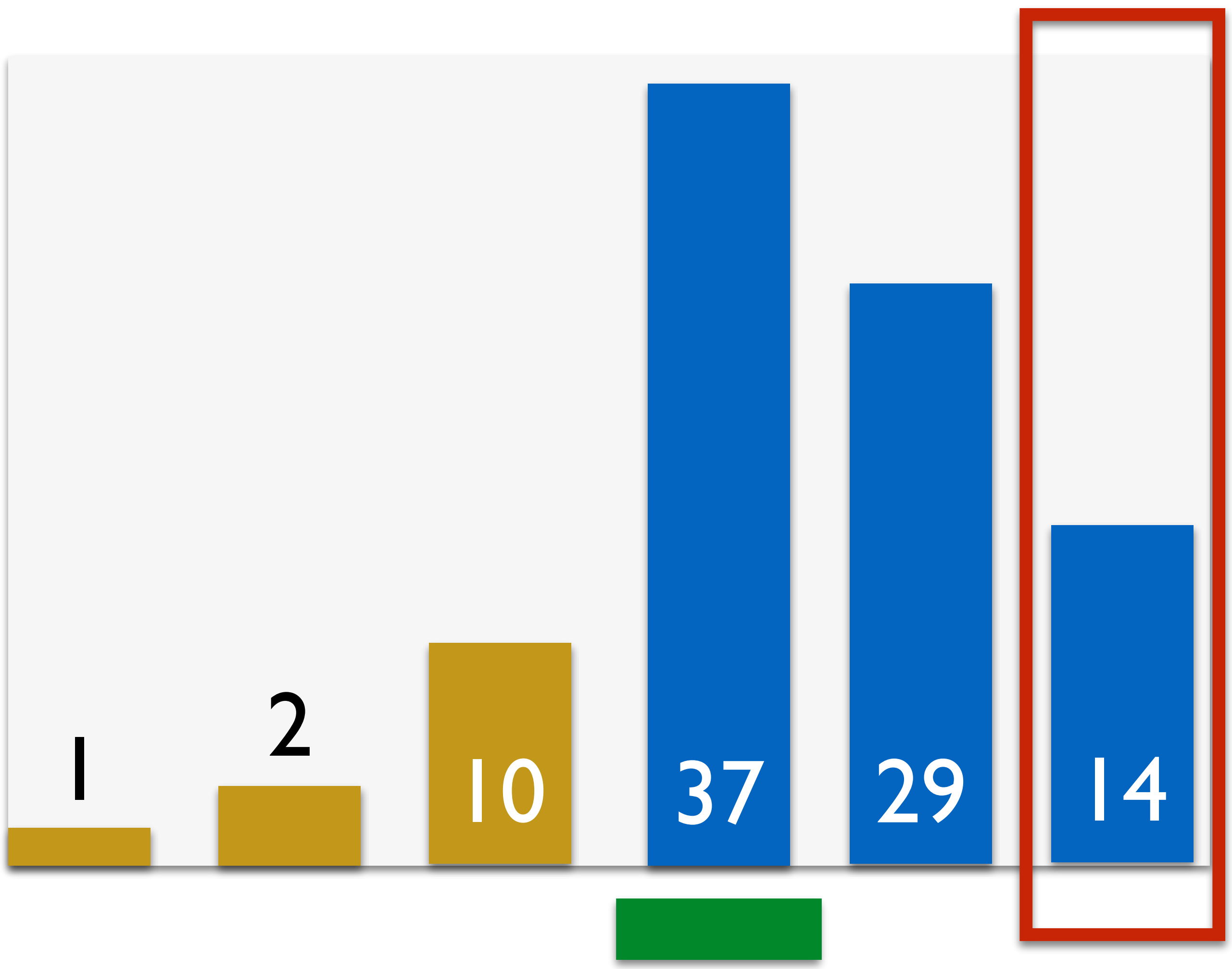

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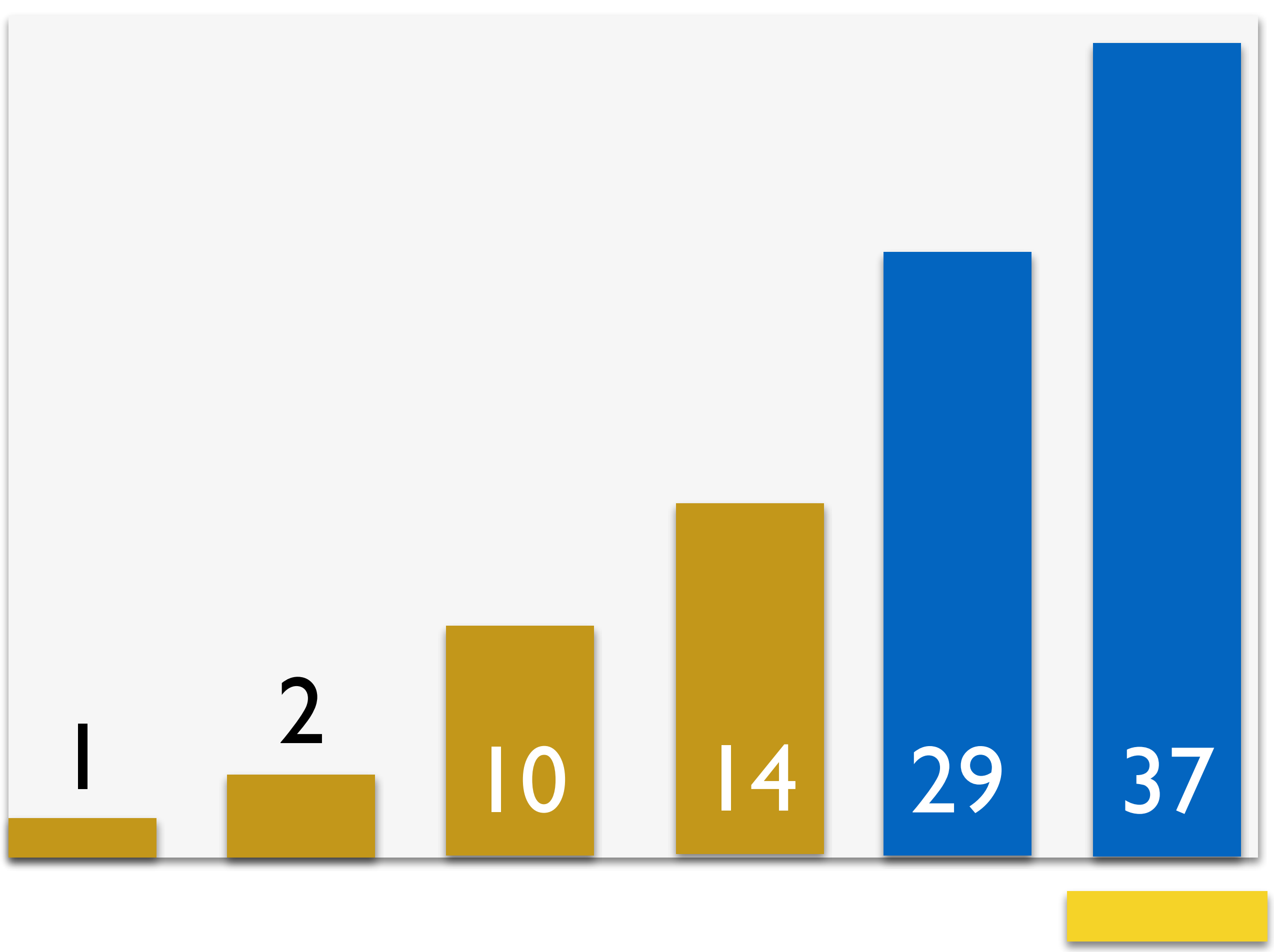

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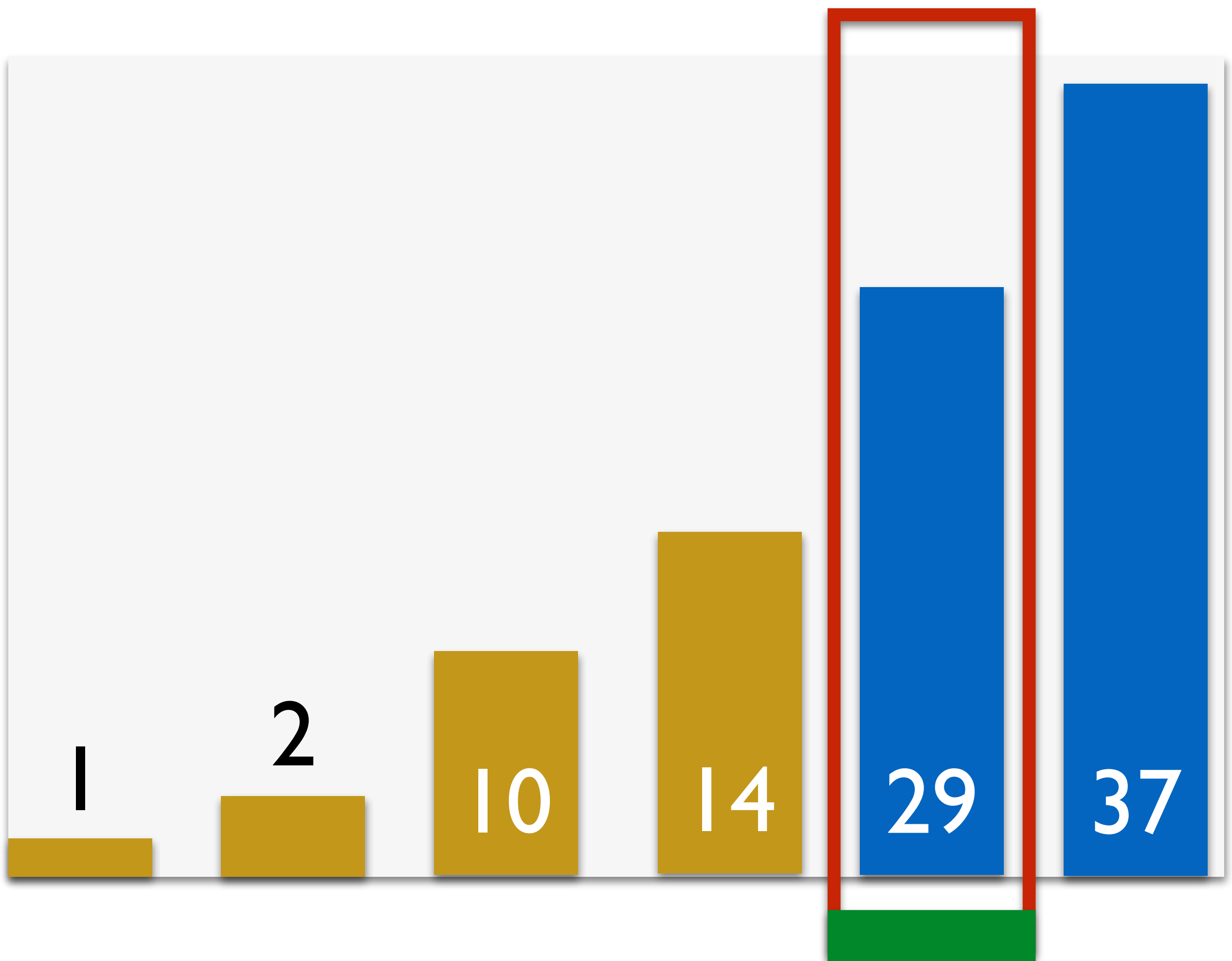

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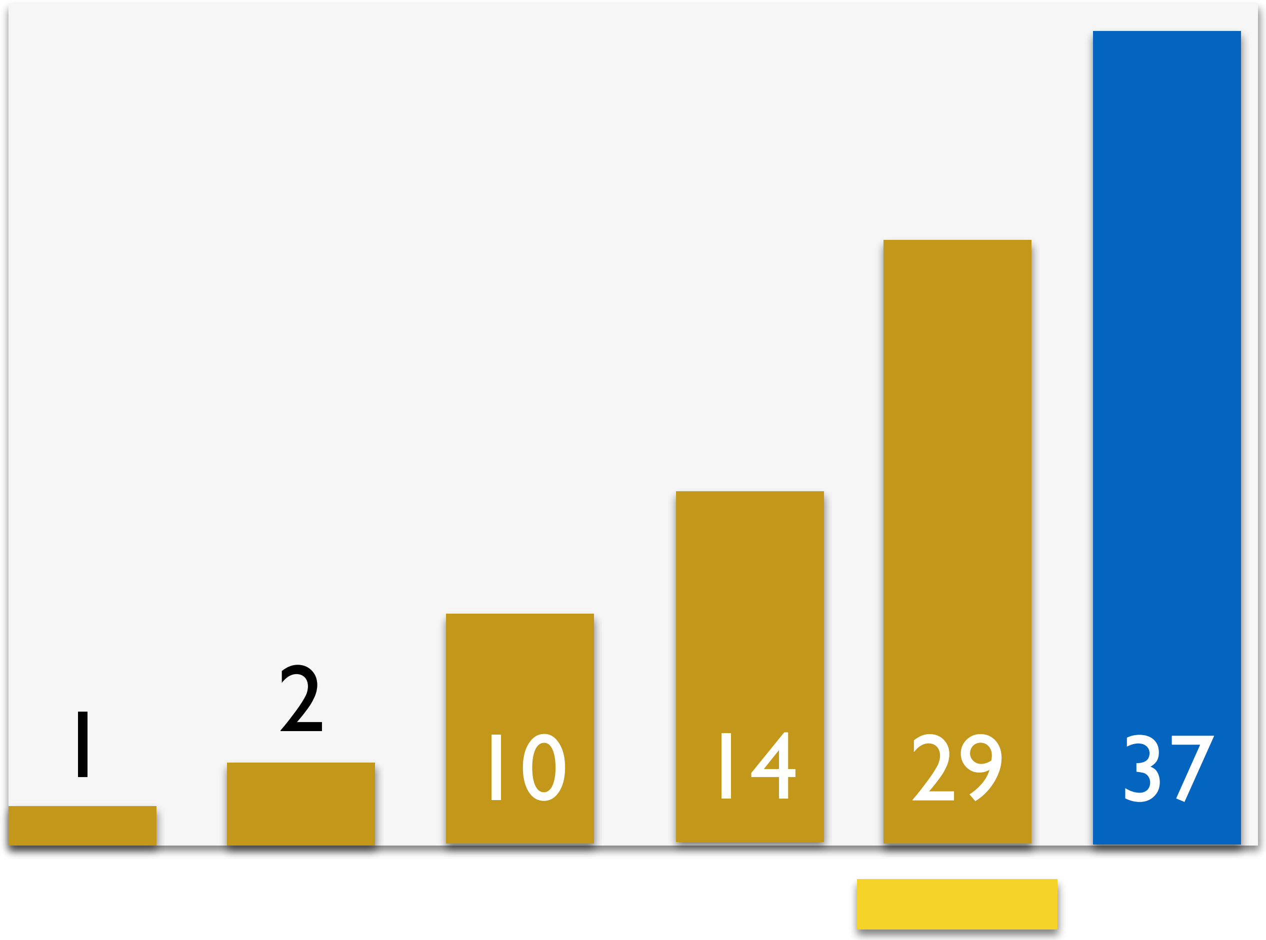

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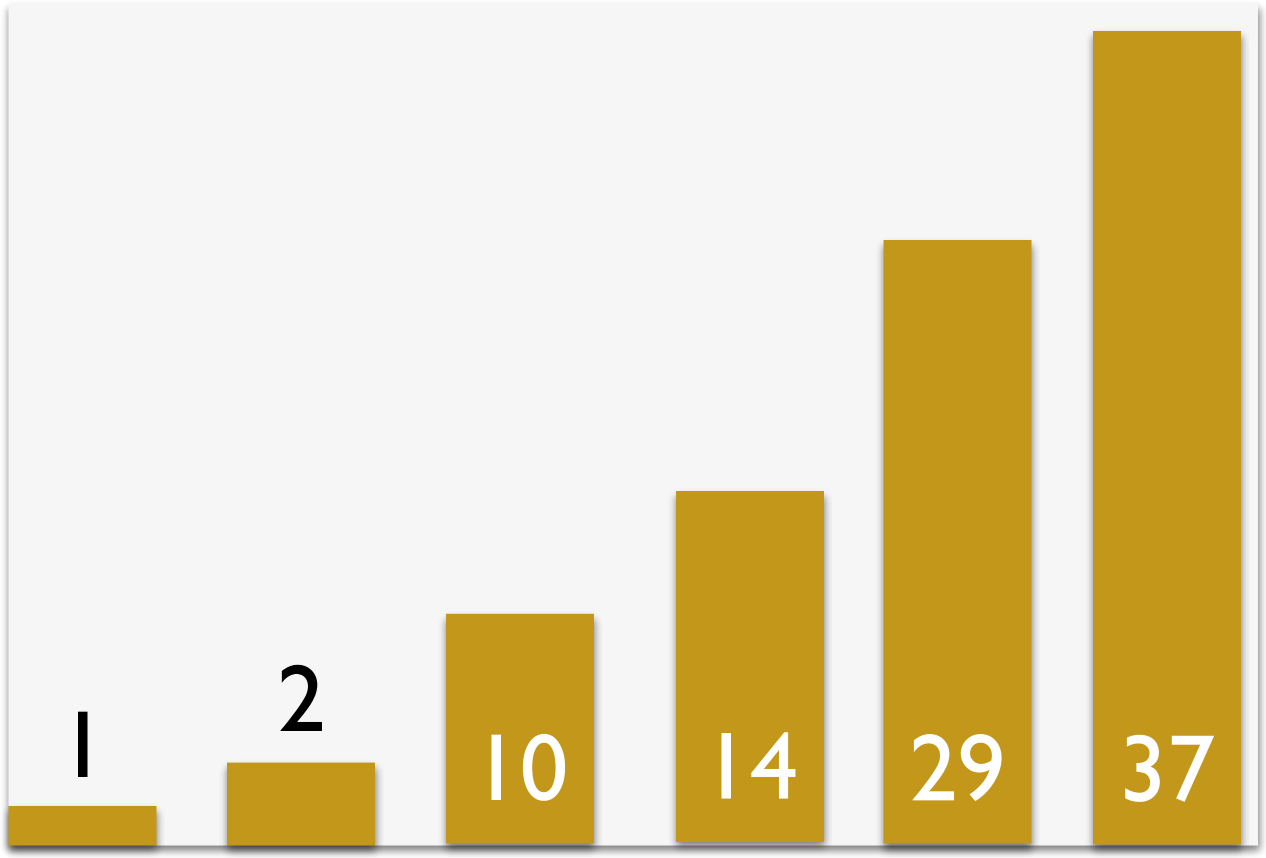

And now we're finally done!

Selection Sort

- Generalize: For each index i in the list `lst`, we need to find the **min** item in `lst[i:]` so we can replace `lst[i]` with that item
- In fact we need to find the position `min_index` of the item that is the minimum in `lst[i:]`
- **Reminder:** how to swap values of variables `a` and `b`?
  - in-line swapping: `a, b = b, a`
- How do we implement this algorithm?

Selection Sort

```
def selection_sort(my_lst):
    """Selection sort of a given mutable sequence my_lst,
    sorts my_lst by mutating it.  Uses selection sort."""


    # find size
    n = len(my_lst)

    # traverse through all elements
    for i in range(n):

        # find min element in the sublist from index i+1 to end

        min_index = get_min_index(my_lst, i)

        # swap min element with current element at i
        my_lst[i], my_lst[min_index] = my_lst[min_index], my_lst[i]
```

You will work on this helper function in Lab 10

Selection Sort

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        # swap min element with current element at i
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```

Even without an implementation, can we guess how many steps does this function need to take?

Selection Sort Analysis

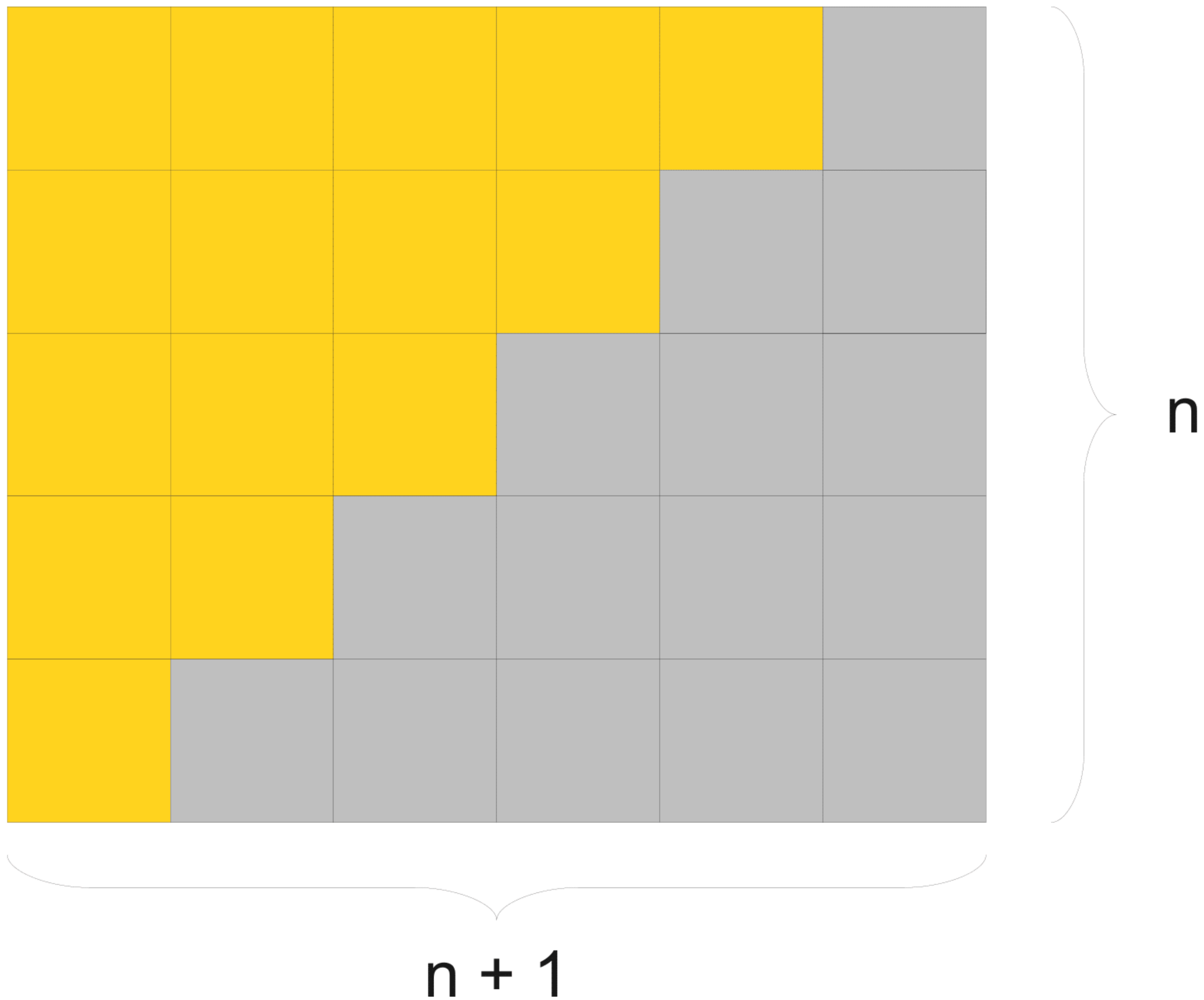
- The helper function `get_min_index` must iterate through index `i` to `n` to find the min item
  - When `i = 0` this is `n` steps
  - When `i = 1` this is `n−1` steps
  - When `i = 2` this is `n−2` steps
  - And so on, until `i = n−1` this is `1` step
- Thus overall number of steps is sum of inner loop steps

$$(n-1)+(n-2)+\cdots+0 \leq n+(n-1)+(n-2)+\cdots+1$$

- What is this sum? (You will see this in MATH 200 if you take it.)

Selection Sort Analysis: Visual

$$n + (n-1) + \ldots + 2 + 1 = n(n+1) / 2$$



Selection Sort Analysis: Algebraic

$$S = n + (n-1) + (n-2) + \cdots + 2 + 1$$

$$+ \quad S = 1 + 2 + \cdots + (n-2) + (n-1) + n$$

$$2S = (n+1) + (n+1) + \cdots + (n+1) + (n+1) + (n+1)$$

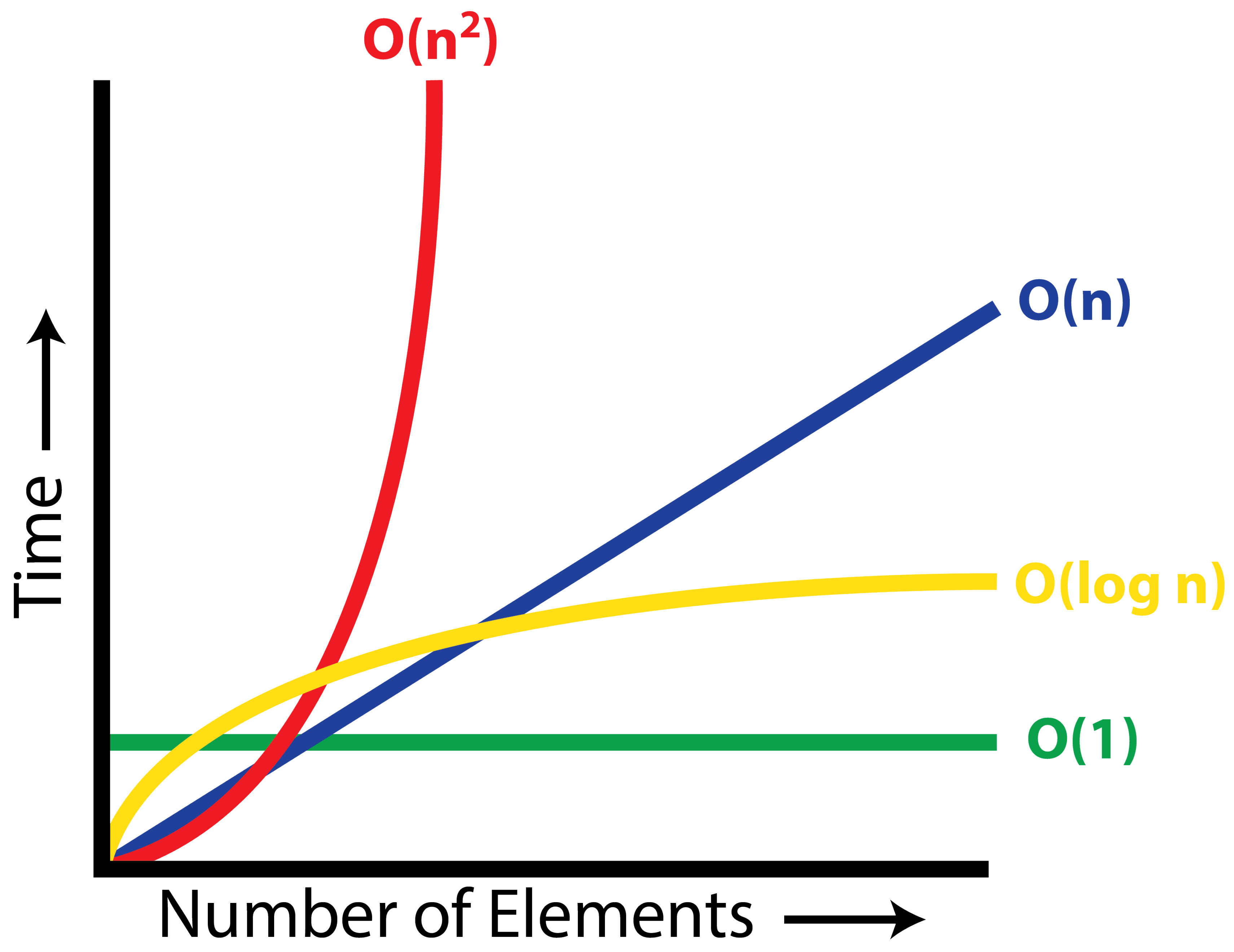
$$2S = (n+1) \cdot n$$

$$S = (n+1) \cdot n \cdot 1/2$$

- Total number of steps taken by selection sort is thus:
  - $O(n(n+1)/2) \; = O(n(n+1)) \; = O(n^2 + n) \; = O(n^2)$

How Fast Is Selection Sort?

- Selection sort takes approximately n^2 steps!



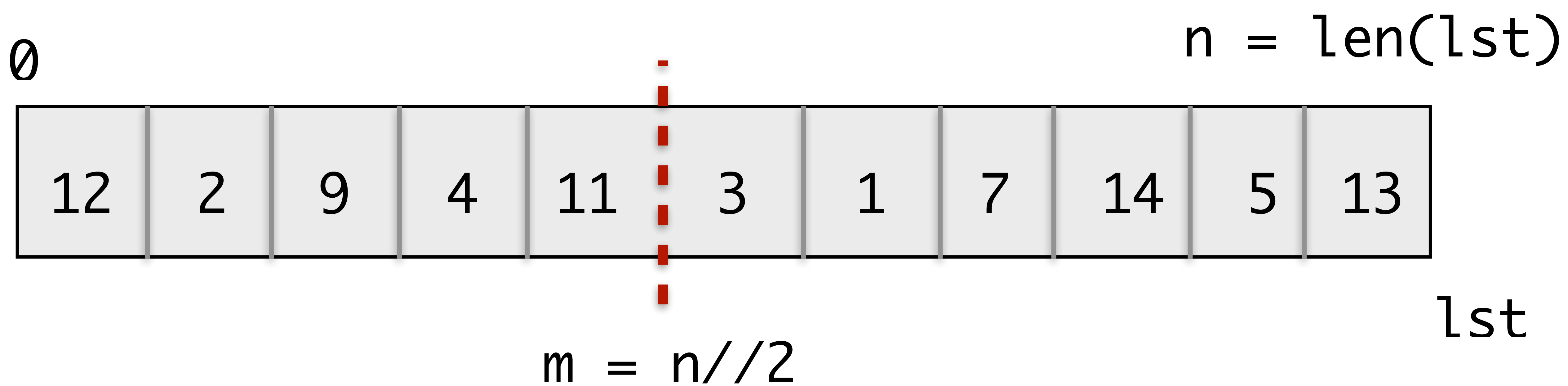
More Efficient Sorting:
Merge Sort

Towards an $O(n \log n)$ Algorithm

- There are other sorting algorithms that compare and rearrange elements in different ways, but are still $O(n^2)$ steps
  - Any algorithm that takes n steps to move each item n positions (in the worst case) will take at least $O(n^2)$ steps
  - To do better than n^2, we need to move an item in fewer than n steps
- We can sort in $O(n \log n)$ time if we are clever: ***Merge sort algorithm*** (Invented by John von Neumann in 1945)

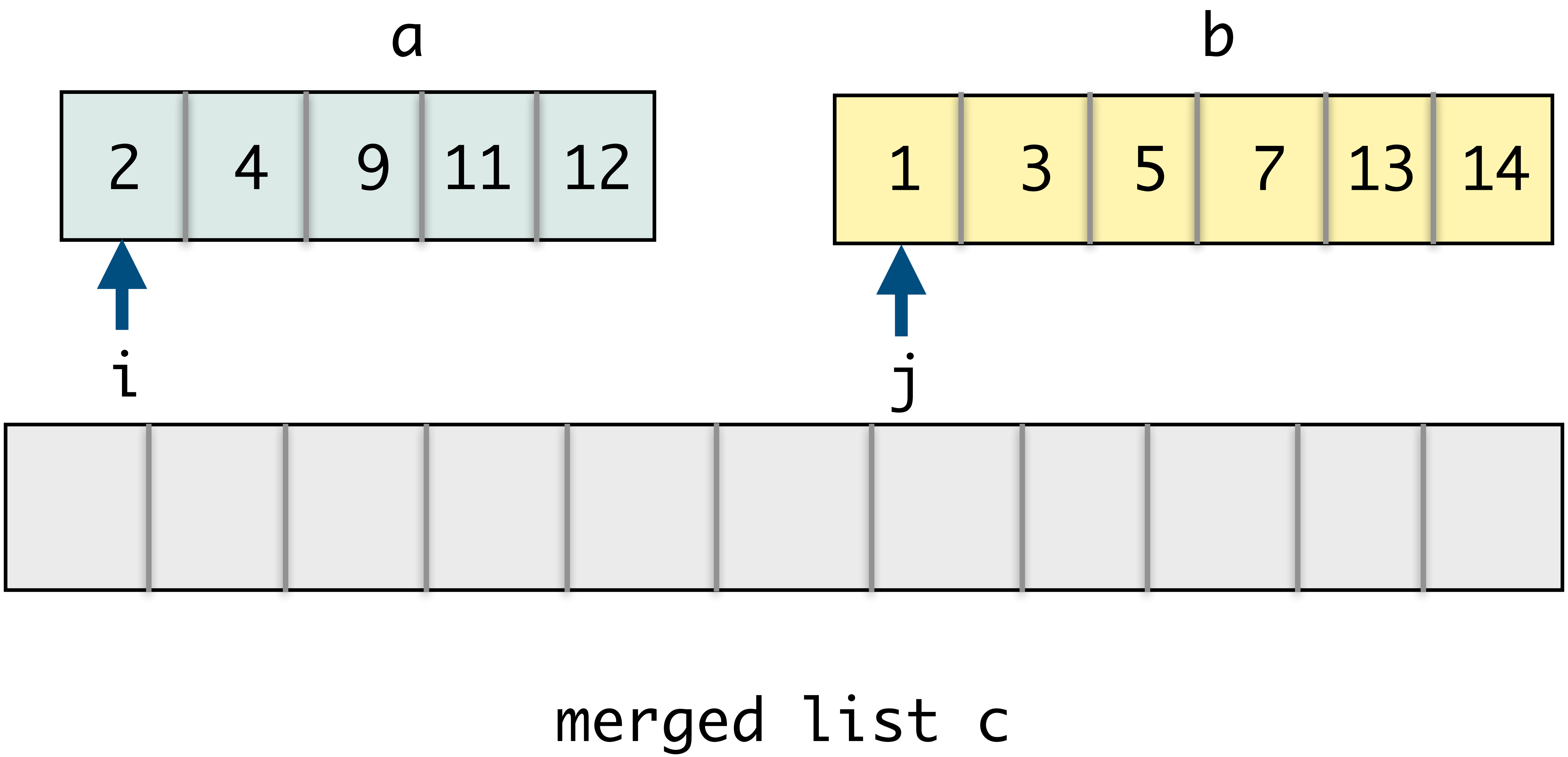
Merge Sort: Basic Idea

- If we split the list in half, sorting the left and right half are smaller versions of the same problem
- **Algorithm:**
  - **(Divide)** Recursively sort left and right half ($O(\log n)$)
  - **(Unite)** Merge the sorted halves into a single sorted list ($O(n)$)



Merging Sorted Lists

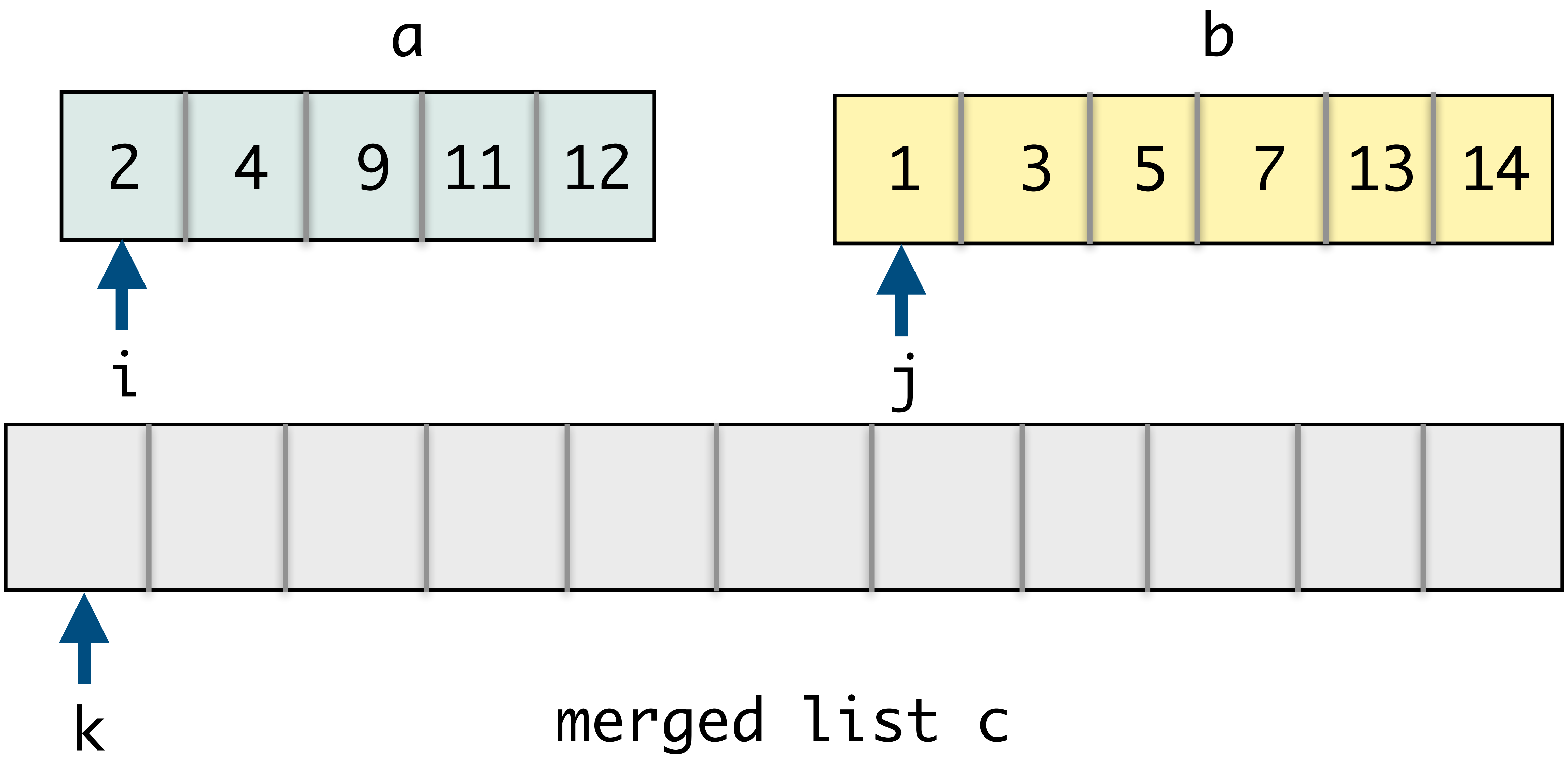
- **Problem.** Given two sorted lists a and b, how quickly can we merge them into a single sorted list?



Merging Sorted Lists

Is `a[i] <= b[j]` ?

- Yes, `a[i]` appended to `c`
- No, `b[j]` appended to `c`



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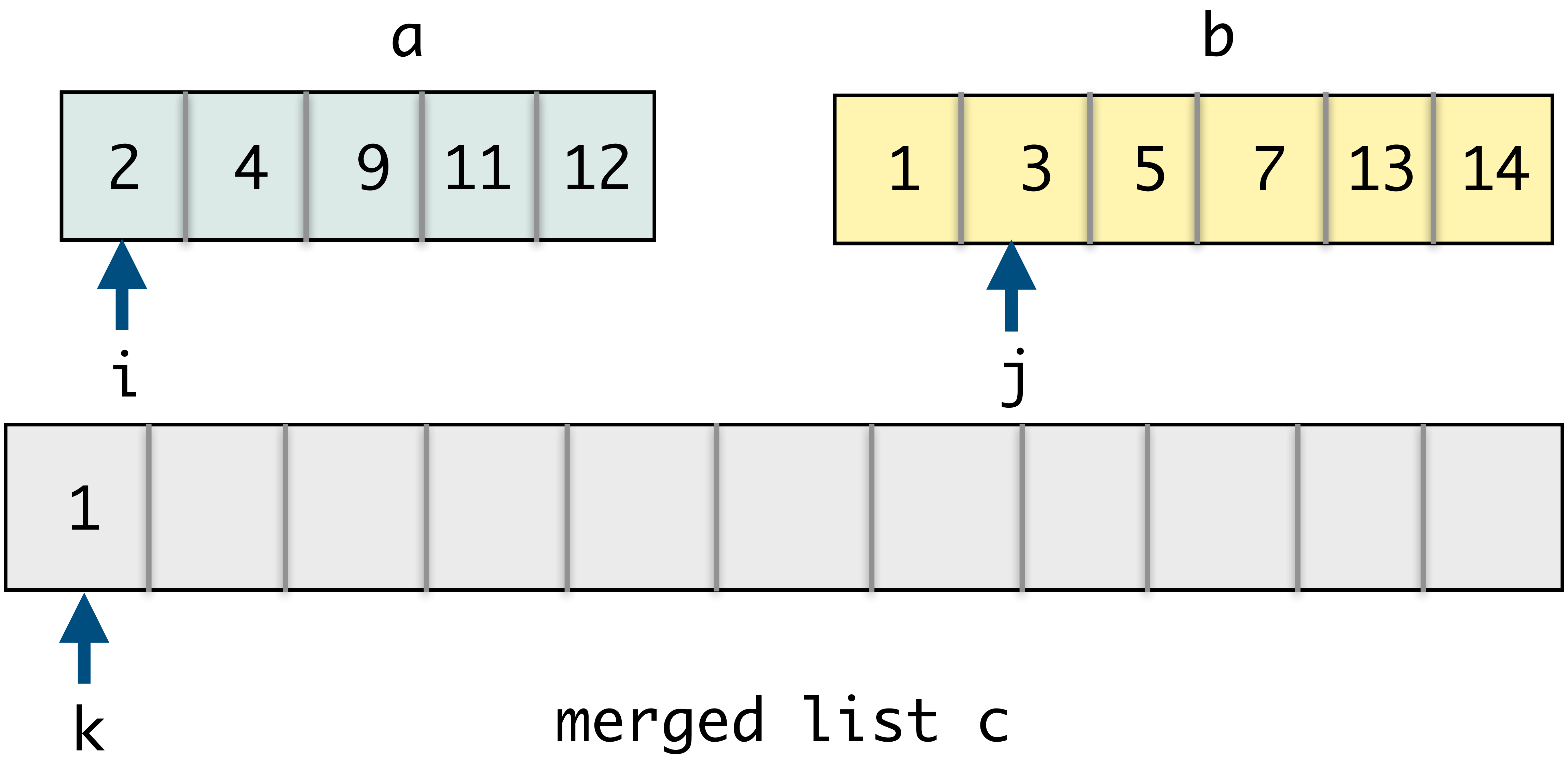

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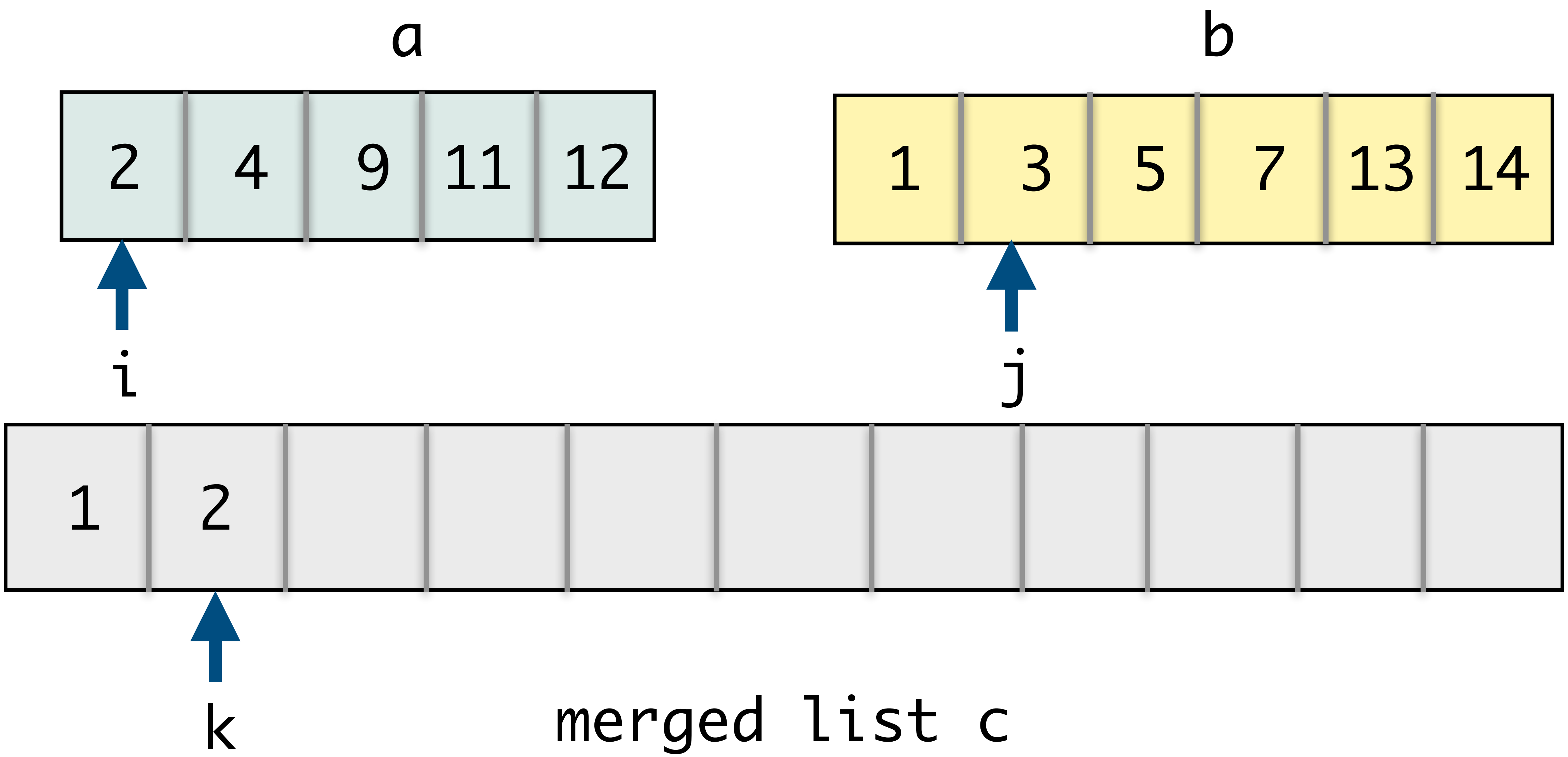

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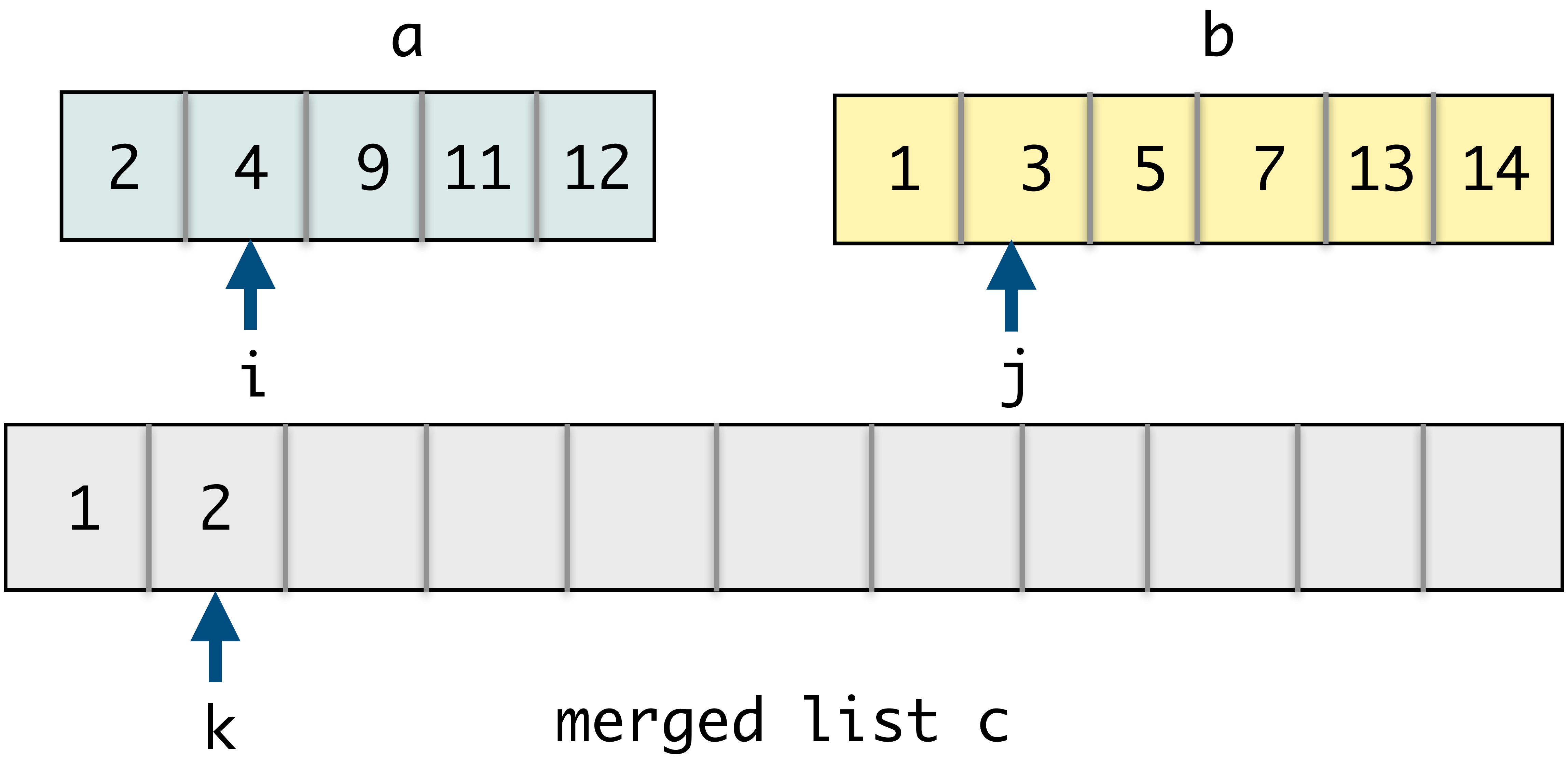

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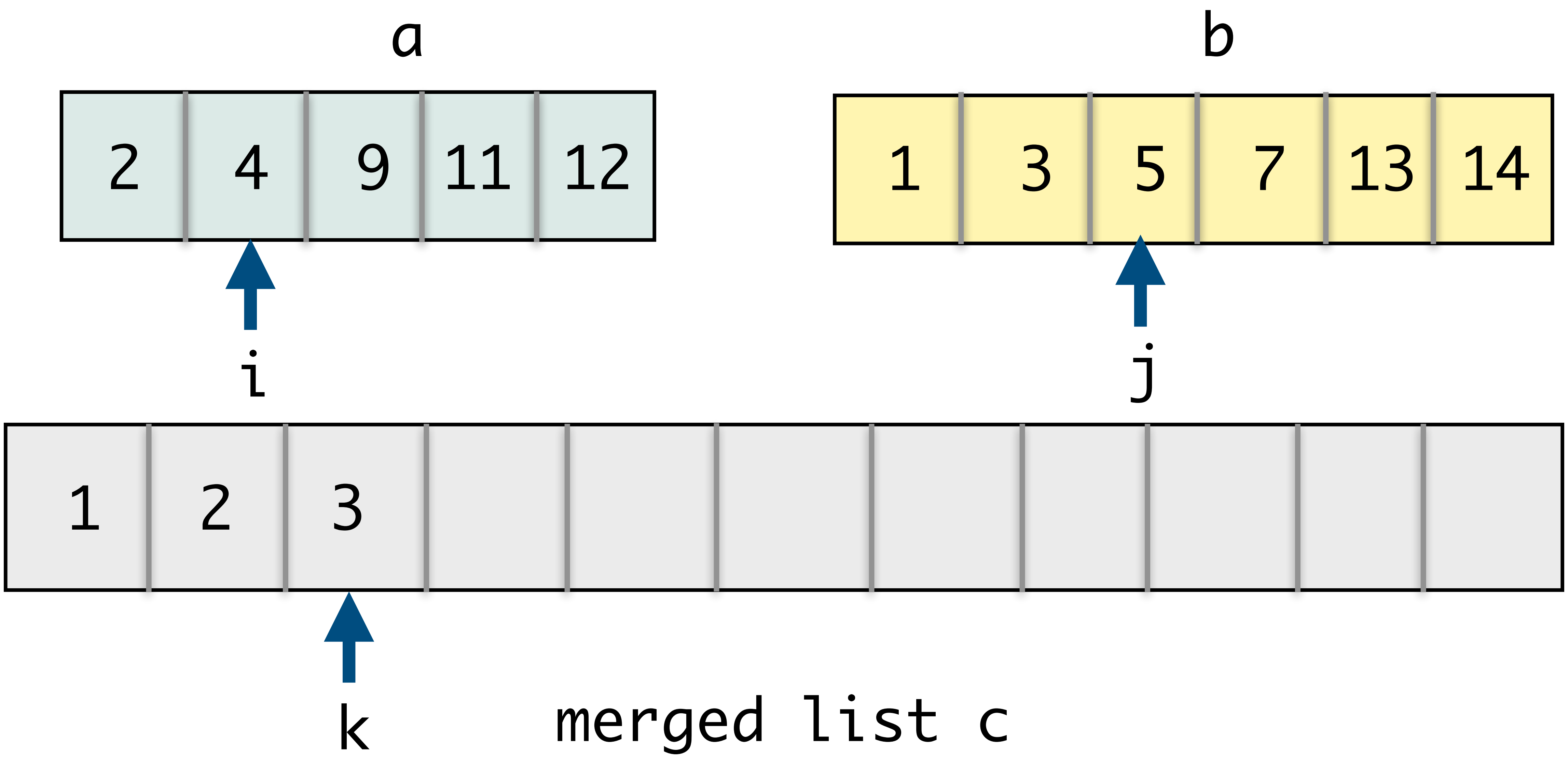

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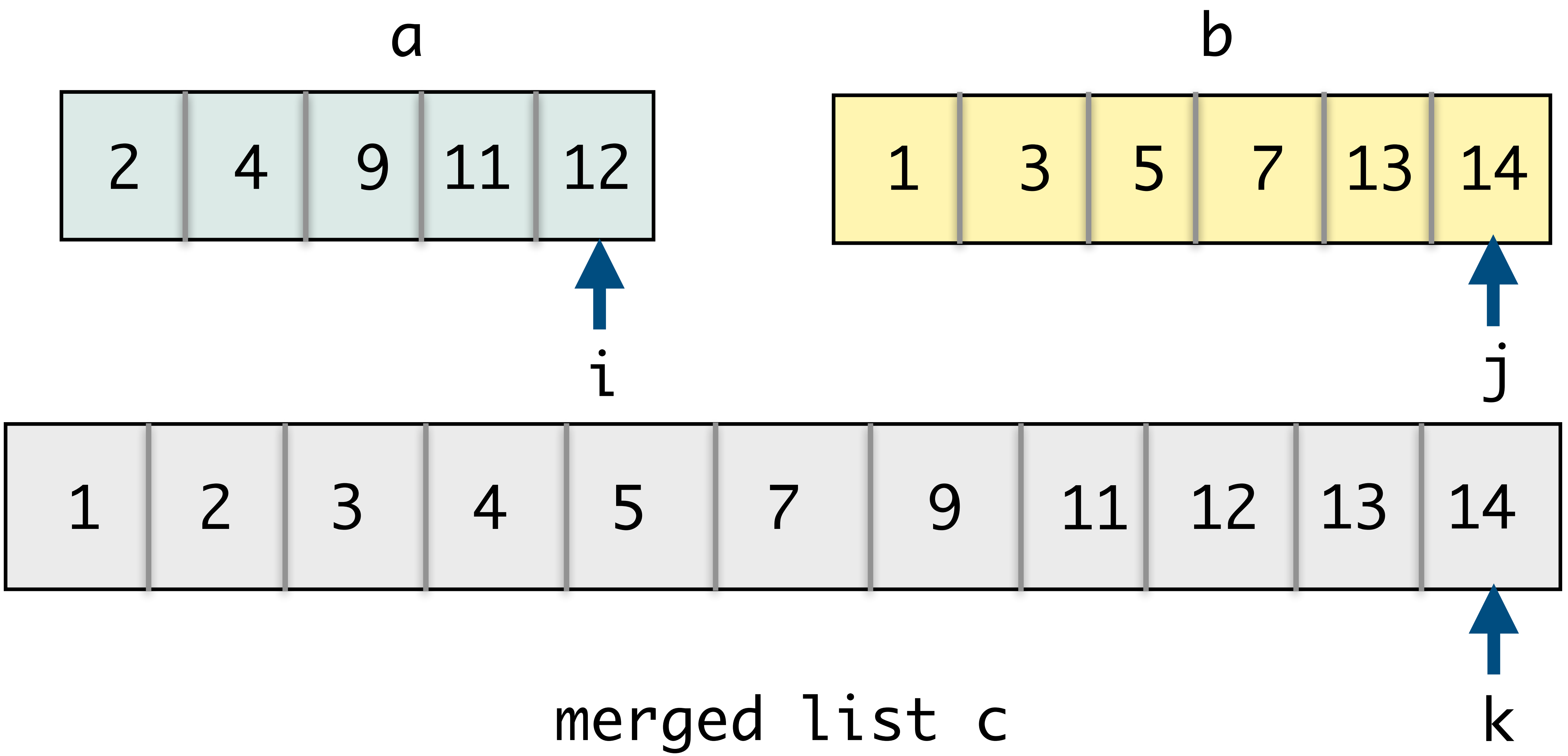


Merging Sorted Lists

- Walk through lists a, b, c maintaining current position of indices i, j, k
- Compare $a[i]$ and $b[j]$, whichever is smaller gets put in the spot of $c[k]$
- Merging two sorted lists into one is an $O(n)$ step algorithm!
- Can use this merge procedure to design our recursive merge sort algorithm!

```
def merge(a, b):
    """Merges two sorted lists a and b,
    and returns new merged list c"""
    # initialize variables
    i, j, k = 0, 0, 0
    len_a, len_b = len(a), len(b)
    c = []
    # traverse and populate new list
    while i < len_a and j < len_b:

        if a[i] <= b[j]:
            c.append(a[i])
            i += 1
        else:
            c.append(b[j])
            j += 1

    # handle remaining values
    if i < len_a:
        c.extend(a[i:])

    elif j < len_b:
        c.extend(b[j:])

    return c
```

Merge Sort Algorithm

- **Base case:** If list is empty or contains a single element: it is already sorted
- **Recursive case:**
  - Recursively sort left and right halves
  - Merge the sorted lists into a single list and return it
- **Question:**
  - Where is the ***sorting*** actually taking place?

```
def merge_sort(lst):
    """Given a list lst, returns
    a new list that is lst sorted
    in ascending order."""
    n = len(lst)

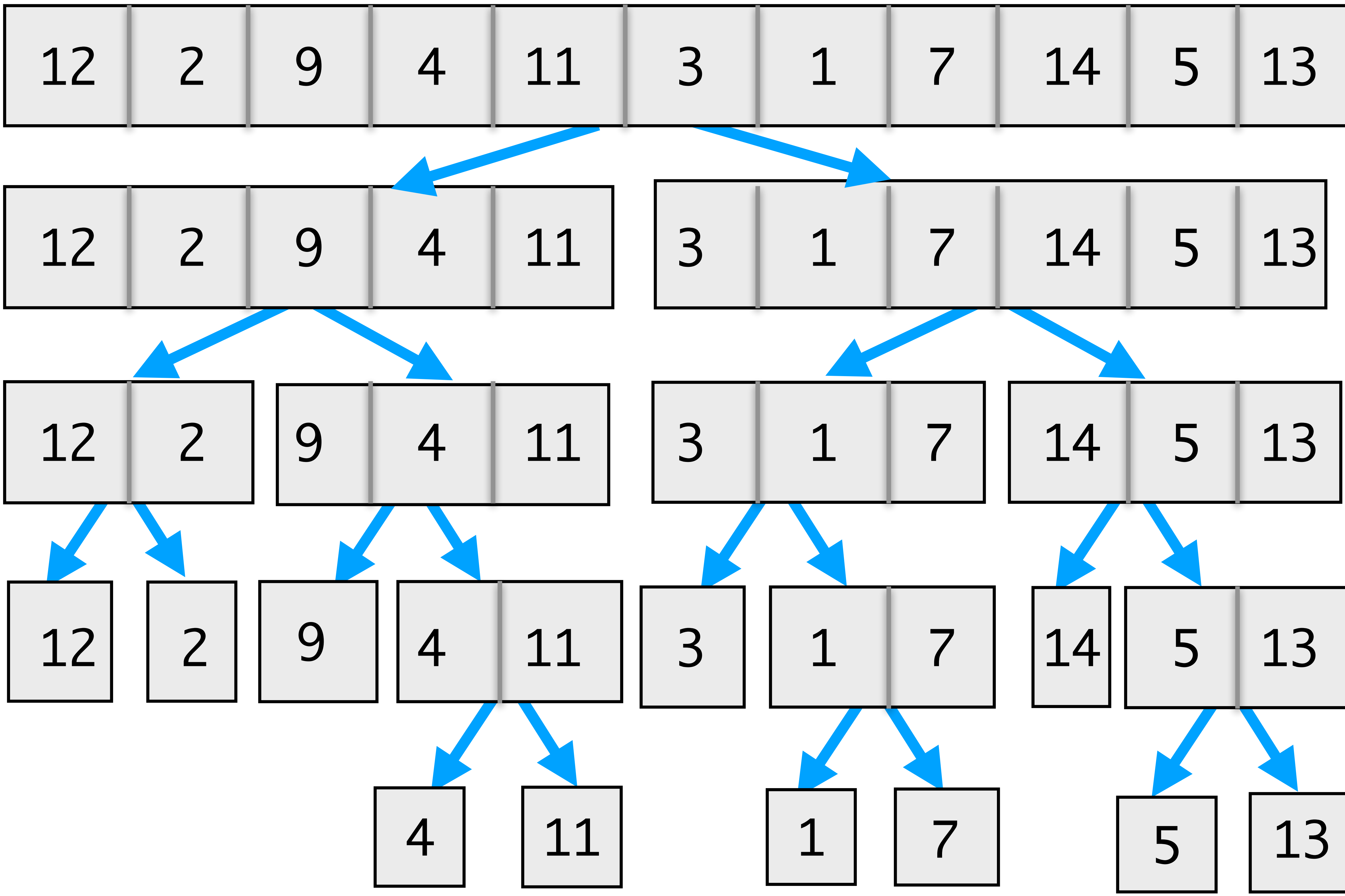
    # base case
    if n == 0 or n == 1:
        return lst

    else:
        m = n//2 # middle

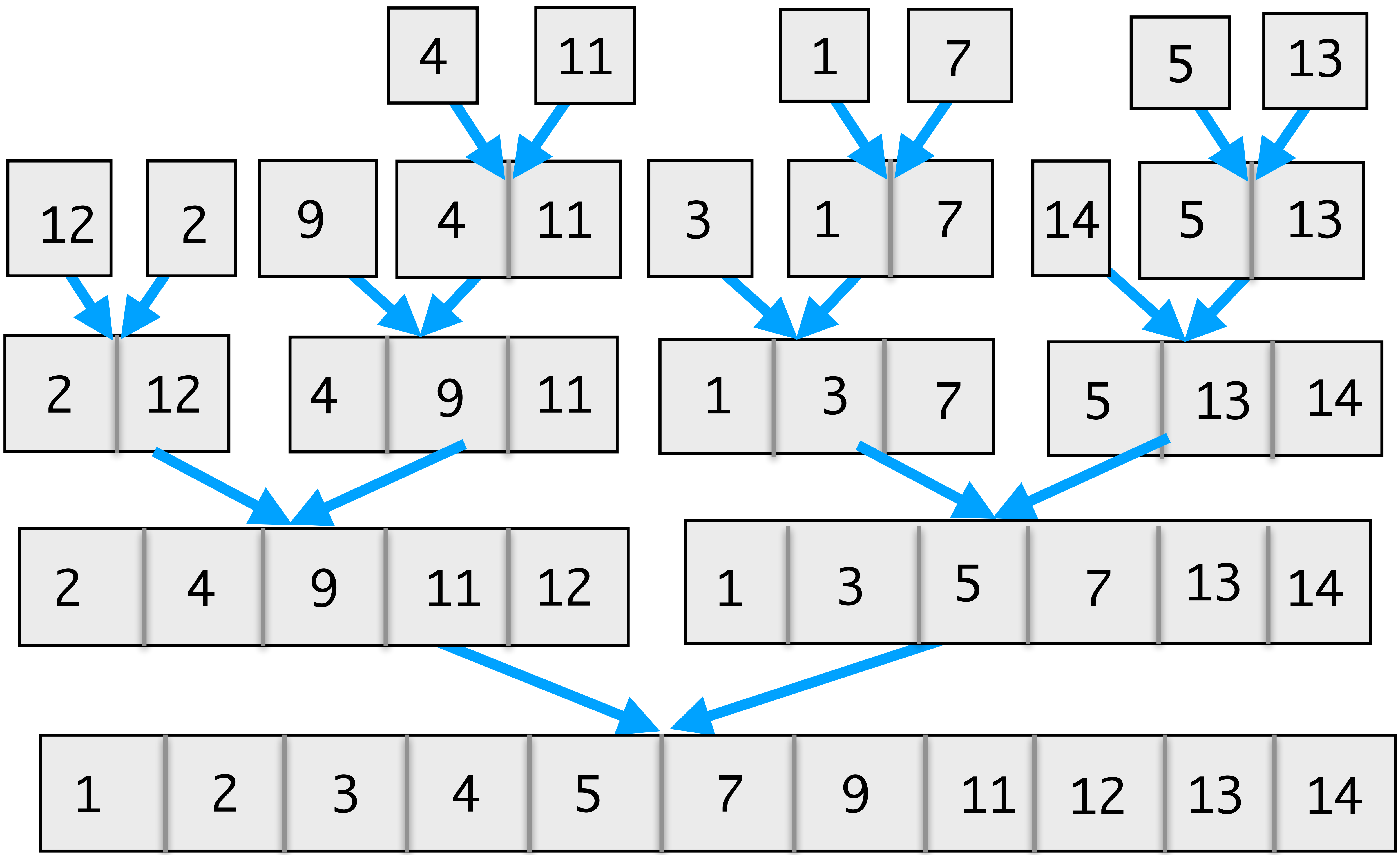
        # recurse on left & right half
        sort_lt = merge_sort(lst[:m])
        sort_rt = merge_sort(lst[m:])

        # return merged list
        return merge(sort_lt, sort_rt)
```

Merge Sort Example

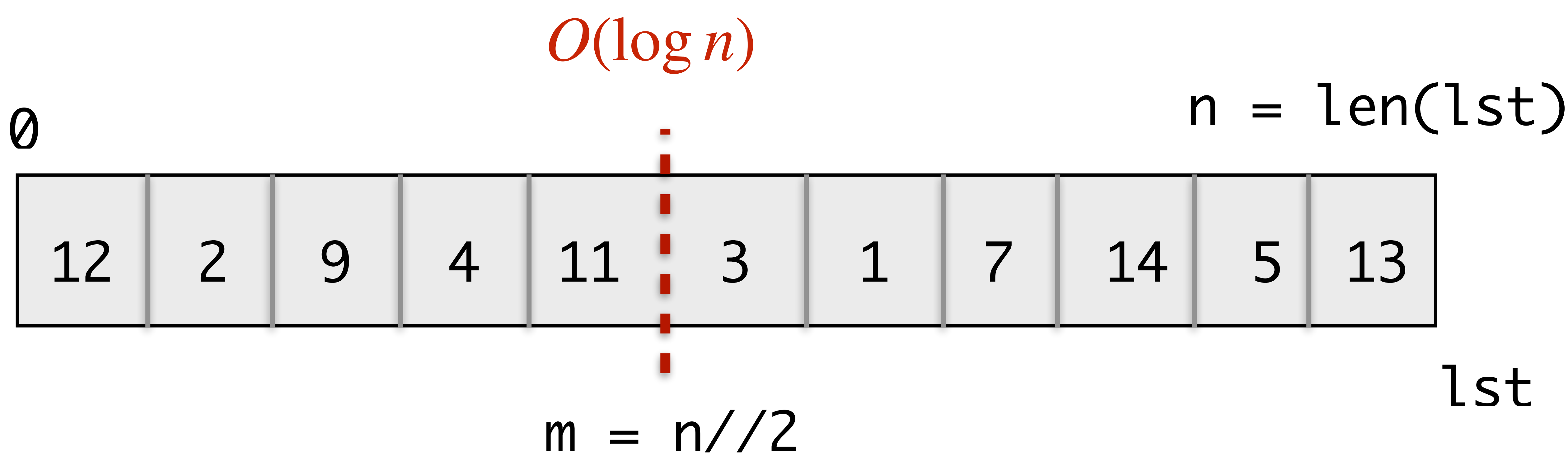


Merge Sort Example



Merge Sort: Basic Idea

- If we split the list in half, sorting the left and right half are smaller versions of the same problem
- **Algorithm:**
  - **(Divide)** Recursively sort left and right half ($O(\log n)$)
  - **(Unite)** Merge the sorted halves into a single sorted list ($O(n)$)



Big Oh Comparisons

- Selection sort: $O(n^2)$
- Merge sort: $O(n \log n)$